CHAPTER 6

PHOTOVOLTAIC SYSTEMS

6.1 INTRODUCTION

The focus of this chapter is on the analysis and design of photovoltaic (PV) systems in their four most commonly encountered configurations. One dichotomy is based on whether the systems are grid connected or not. Grid-connected systems, in turn, can be broken into two categories based on which side of the electric meter the PVs are placed. Relatively small-scale systems, usually on rooftops, feed power directly to customers on their side of the meter. The grid is used as a backup buffer supply. These "behind-the-meter" systems compete against the retail price of grid electricity, which helps their economics. Systems on the utility side of the meter are generally much larger and their owners sell power into the wholesale electricity market. These systems are more likely to use single- or double-axis tracking systems, with or without concentrated sunlight, than rooftop systems that dominate behind-the-meter systems.

Off-grid systems can also be broken into two categories. One describes stand-alone systems that typically use batteries for energy storage. These range all the way from "pico-scale" PV-powered lanterns and cell phone chargers to larger solar-powered homes, schools, and small businesses, especially in emerging economies around the globe. The other is for loads that are directly connected to the PVs with no intermediate electronics or battery storage. These are mostly water-pumping systems for which water storage replaces the need for electricity storage. Such systems can be exceedingly simple and reliable, but are surprisingly tricky to analyze as we shall see in this chapter.

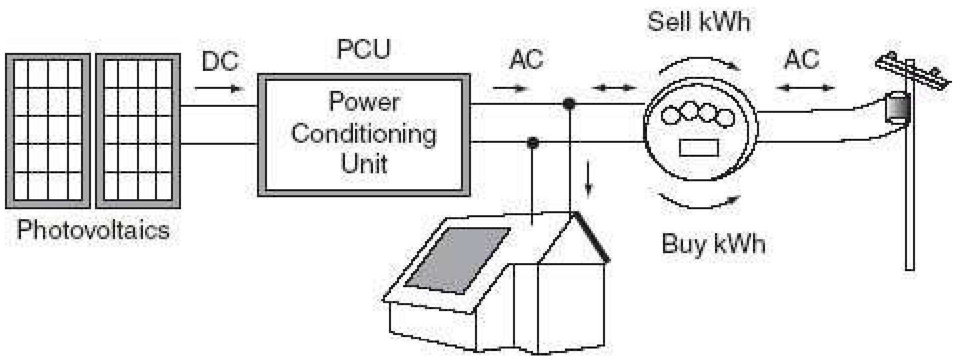
6.2 BEHIND-THE-METER GRID-CONNECTED SYSTEMS

Grid-connected PV systems on the customer's side of the meter have a number of desirable attributes. Compared to utility-scale systems, they avoid land acquisition costs since they are located on the owner's property, and they compete against the much higher retail price of electricity. Compared to off-grid PV systems, they are less expensive since they avoid the cost and inefficiencies, as well as reliability aspects, of batteries and backup generators. Off-grid PV systems, on the other hand, typically compete against much more expensive fuel-fired generators rather than relatively cheap utility power.

6.2.1 Physical Components in a Grid-Connected System

Figure 6.1 shows a simplified diagram of a grid-connected PV system. The customer, in this case, is shown as a single-family residence for which a typical PV system may be rated at something like 1–10 kW of generation capacity. A similar system on a commercial building might have tens of kW to perhaps a megawatt or two of capacity, typically located on rooftops and perhaps in parking lots. While residential and commercial systems are physically similar, the differences in utility rate structures and financial incentives for residential and commercial facilities lead to significantly different economics. In this section, we will deal with the systems.

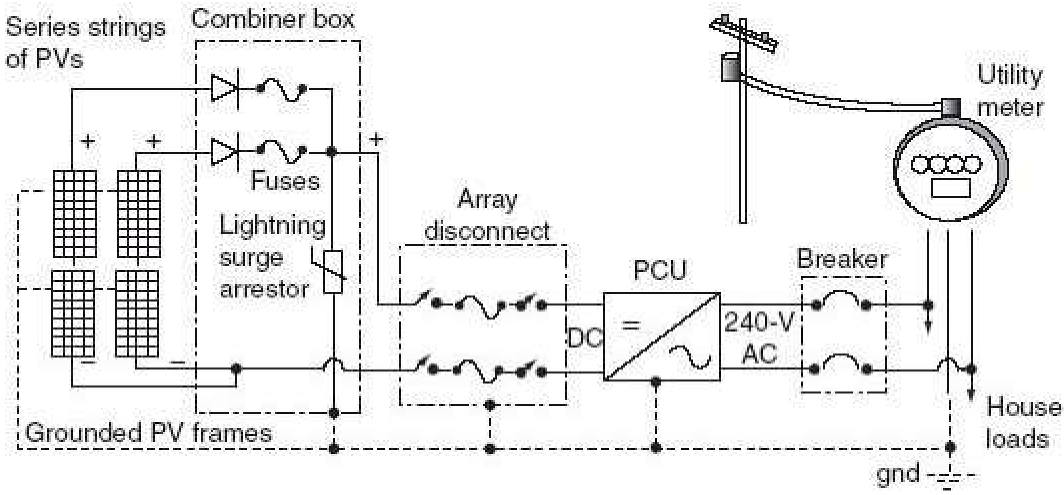
FIGURE 6.1 Simplified grid-connected PV system with net metering.



The PVs in Figure 6.1 deliver DC power to a power-conditioning unit (PCU). The PCU includes a maximum power point tracker (MPPT) to keep the PVs operating at the most efficient point on their *I–V* curve as well as an inverter to convert DC to AC. If the PVs supply less than the immediate demand of the building, the PCU draws supplementary power from the utility grid; so building demand is always satisfied. If, at any moment, the PVs supply more power than is needed, the excess is sent back onto the grid, potentially spinning the electric meter backward building up an energy credit with the utility. The system is relatively simple since failure-prone batteries are not needed for backup power—although, sometimes they may be included if utility outages are problematic.

Some details of the various components in a typical grid-connected, home-size system are shown in Figure 6.2. The system consists of the array itself with leads from each string sent to a combiner box that includes blocking diodes, individual fuses for each string, and usually a lightning surge arrestor. Heavy gauge wire from the combiner box delivers DC power to a fused array disconnect switch, which allows the PVs to be completely isolated from the system. The PCU includes an MPPT, a DC-to-AC inverter, a ground-fault circuit interrupter (GFCI) that shuts the system down if any currents flow to ground, and circuitry to disconnect the PV system from the grid if utility power is lost. The PCU sends AC power, usually at 240 V, through a breaker to the utility service panel. By tying each end of the inverter output to opposite sides of the service panel, 120-V power is delivered to each household circuit.

FIGURE 6.2 Principal components in a grid-connected PV system using a single inverter and a single utility meter.



The PCU must be designed to quickly and automatically drop the PV system from the grid in the event of a utility power outage. When there is an outage, breakers automatically isolate a section of the utility lines in which the fault has occurred, creating what is referred to as an "island." A number of very serious problems may occur if, during such an outage, a self-generator, such as a grid-connected PV system, supplies power to that island.

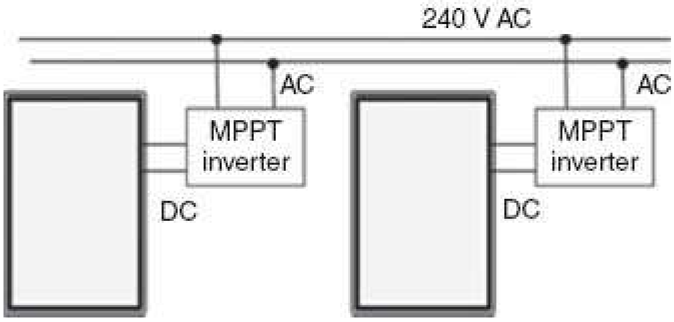
Most faults are transient in nature, such as a tree branch brushing against the lines, and so utilities have automatic procedures that are designed to limit the amount of time the outage lasts. When there is a fault, breakers trip to isolate the affected lines, and then they are automatically reclosed a second or two later. It is hoped that in the interim the fault clears and customers are without power for just a brief moment. If that does not work, the procedure is repeated with somewhat longer intervals until finally, if the fault does not clear, workers are dispatched to the site to take care of the problem. If a self-generator is still on the line during such an incident, even for less than one second, it may interfere with the automatic reclosing procedure leading to a longer-than-necessary outage. And if a worker attempts to fix a line that has supposedly been disconnected from all energy sources, but it is not, then a serious hazard has been created.

When a grid-connected system must provide power to its owners during a power outage, a small battery backup system may be included. If the users really need uninterruptible power for longer periods of time, the battery system can be augmented with a generator.

6.2.2 Microinverters

An alternative approach to the single inverter system shown in Figure 6.2 is based on each PV module having its own microinverter/MPPT mounted directly onto the backside of the panel (Fig. 6.3). These microinverters offer several significant advantages. With each module having its own MPPT, bypass diodes are no longer needed and the risk of a poorly performing module bringing down an entire string (Section 5.8.2) is eliminated. Similarly, having many small inverters avoids the risk of an inverter malfunction taking down the entire array. Individual inverters also facilitate remote monitoring which can help identify performance issues module by module, and if a module needs to be replaced, it can be individually shut down and safely removed without affecting the rest of the array.

FIGURE 6.3 An alternative to having a single MPPT/inverter for an entire array is to provide microinverters for each module.

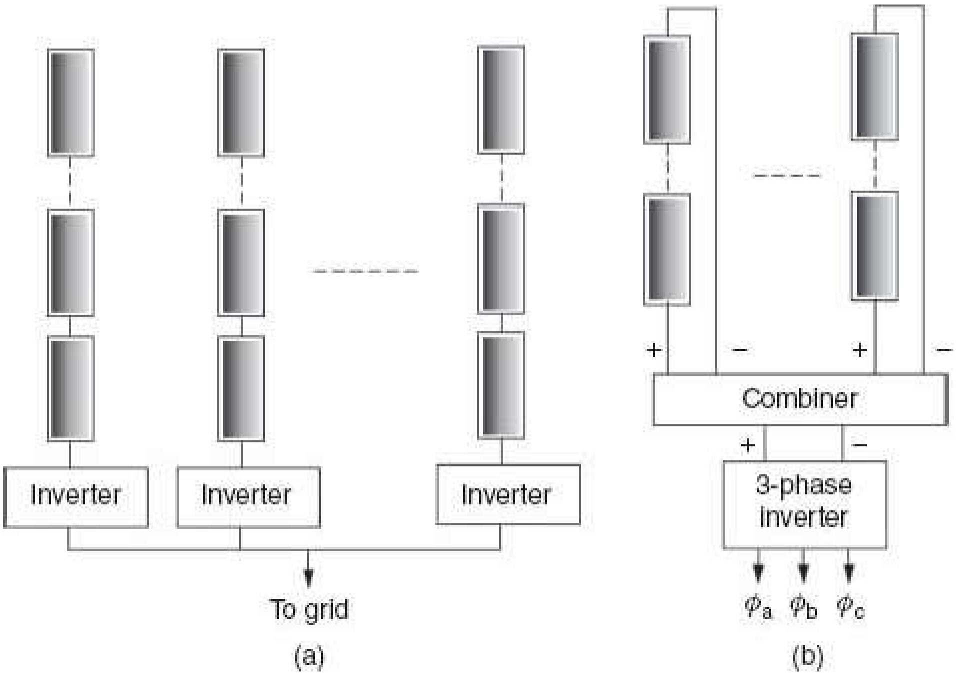


There are some safety advantages to multiple microinverters as well. For example, the array can be wired using conventional AC components at lower voltages than are common in DC systems. AC circuit breakers can be safer and cheaper than their DC counterparts because they are designed to disconnect during the zero-crossing of AC current. With DC there is no zero-crossing and, as we learned in Section 2.7.2, current momentum caused by inductance means it cannot be stopped instantaneously and trying to do so can cause a potentially dangerous arc. Codes now often require special arc-fault circuit interrupters for DC wiring to avoid the risk of fires. Finally, the ability to disconnect modules at the breaker box, or even remotely, reduces the danger of PVs still being energized during emergencies when firefighters, for example, might have to be up on the roof.

Having AC modules also makes fitting an array to an irregular roof surface-–especially one that has potential shading problems—much easier since the designer is not constrained to parallel strings with equal numbers of modules per string. Each module is totally independent and can be put anywhere. In fact, a customer's system can easily be expanded as loads change or as budgets allow.

The above advantages come at a cost. A single inverter may be significantly cheaper than a large number of microinverters—especially for larger arrays. For such systems, strings of PV modules may be tied into inverters in a manner analogous to the individual inverter/module concept (Fig. 6.4a). By doing so, the system is modularized making it easier to service portions of the system without taking the full system offline. Expensive DC cabling is also minimized making the installation potentially cheaper than a large, central inverter. Large, central inverter systems providing three-phase power to the grid are also an option (Fig. 6.4b). Details of how such inverters work were described in Chapter 3.

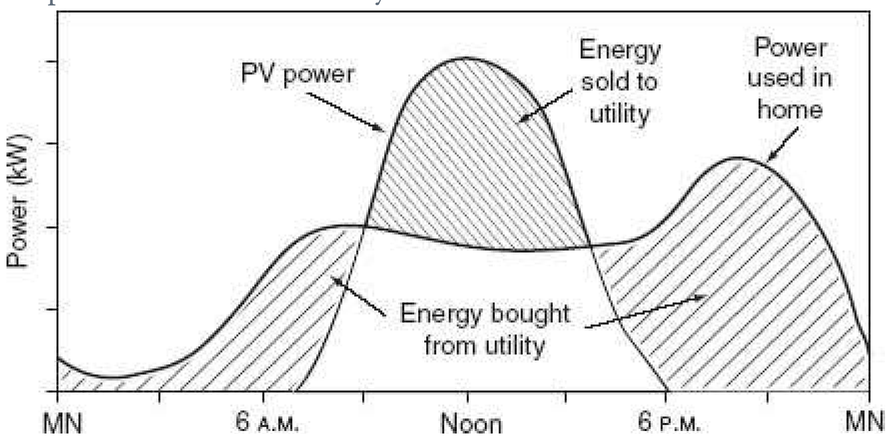
FIGURE 6.4 Larger grid-connected systems may use an individual inverter for each string (a), or may incorporate a large, central inverter system to provide three-phase power (b).



6.2.3 Net Metering and Feed-In Tariffs

The system shown in Figure 6.1, with its single electric meter that spins in both directions, is an example of a *net metering* billing arrangement with the local utility. As shown in Figure 6.5, whenever the PV system delivers more power than the home needs at that moment, the excess spins the electric meter backward, building up a credit with the utility. At other times, when demand exceeds that supplied by the PVs, the grid provides supplementary power. The customer's monthly electric bill is only for the net amount of energy that the PV system is unable to supply.

FIGURE 6.5 During the day, any excess power from the array is sold to the utility; at night, power is purchased from the utility.



In its simplest form, net metering requires no new equipment since essentially all electricity meters run in either direction. The financial accounting, on the other hand, can be rather complex. For example, in an effort to encourage customers to shift their loads away from peak demand times, some offer residential time-of-use (TOU) rates. For many utilities, the peak demand occurs on hot, summer afternoons when air conditioners are humming and less cost-effective reserve power plants are put on line, so it makes sense for them to try to charge more during those times. Conversely, at night when there is idle capacity, rates can be significantly lower. Customers with PVs who chose TOU rates may be able to sell power to the utility at a higher price during the day than they pay to buy it back at night.

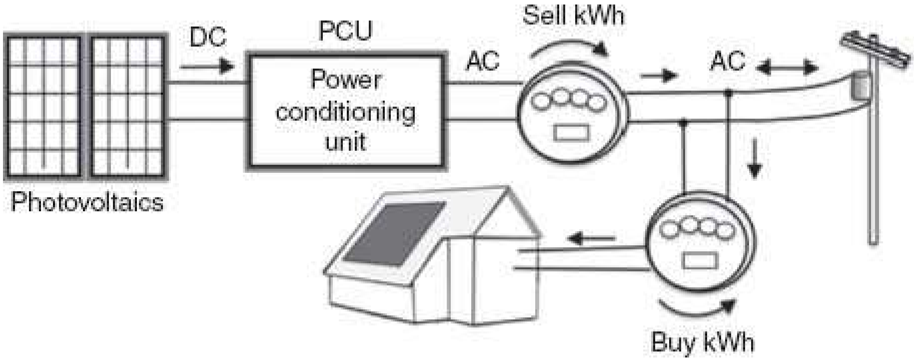
Table 6.1 illustrates an example in which a PV system provides all of the electricity needed by a household during a bright summer month. If a flat-rate structure were chosen, the household bill would be zero for that month. By signing up for TOU rates, however, the net bill would have the utility owing the customer $24 for that month. Utilities often allow negative kWh sales to a customer over a given time period, but they usually true up their books in such a way that makes sure the utility never ends up owing the customer money over an entire year.

TABLE 6.1 TOU Energy and Dollar Calculations for an Example PV System that Provides 100% of Electricity Demand for an Example Month in the Summer

Period	Time	Rate ($/kWh)	PV Delivers (kWh/mo)	Household Demand (kWh/mo)	Net Utility Usage (kWh/mo)	Bill with Solar and TOU ($/mo)
Partial peak	Morning	0.17	400	300	−100	(17.00)
Peak	Afternoon	0.27	500	400	−100	(27.00)
Off peak	Evening	0.10	100	300	200	20.00
Total			1000	1000	0	(24.00)

It is also possible to use a two-meter system, one to measure all of the power generated by the PVs and the other to measure all of the power used in the building (Fig. 6.6). That allows separate rates to be created for each in what is called a *feed-in tariff*. This approach to renewables, which started in Europe, has begun to be offered by utilities in the United States as well. Feed-in tariffs can be designed to encourage adoption of PVs by guaranteeing at the outset a generous price for all electricity generated by the customer's system. This greatly reduces the uncertainty about the value of the PVs, which makes them that much easier to finance. Each year the utility has the ability to adjust the tariff for all new customers, so that as the cost of new systems decreases over time so can the tariff.

FIGURE 6.6 A two-meter system allows a feed-in tariff to provide separate rates for power generated by PVs and power used by customers.



6.3 PREDICTING PERFORMANCE

PV modules are rated under standard test conditions (STCs) that include a solar irradiance of 1 kW/m^2 (called "1-sun"), a cell temperature of 25°C, and an air mass ratio of 1.5 (AM1.5). Under these laboratory conditions, module outputs are thus often referred to as "watts STC" or just "peak watts" (Wp). Out in the field, modules are subject to very different conditions and their outputs will vary significantly from the STC rated power that the manufacturer specifies. Insolation is not always 1-sun, modules get dirty, and cells are typically 20–40°C hotter than the surrounding air. Unless it is very cold outside, or it is not a very sunny day, cells will usually be much hotter than the 25°C at which they are rated.

6.3.1 Nontemperature-Related PV Power Derating

A simple way to deal with the goal of converting STC ratings into expected AC power delivered under real field conditions is to introduce a derating factor:

(6.1) $$P_{AC} = P_{DC,STC} \times \text{Derate factor}$$

To that end, Sandia National Laboratory has created a performance evaluation model called the Solar Advisor Model (SAM) that is the basis for a now commonly used online PV performance calculator called PVWATTS. PVWATTS is readily available on the National Renewable Energy Laboratory (NREL) website. Their calculator provides a number of estimates of factors that can contribute to an overall derate factor, including default values that can be modified by users to suit their own circumstances. Table 6.2 presents their estimates.

TABLE 6.2 PVWATTS Derate Factors for DC-STC to AC Power Ratings (Not Including Temperature Impacts)

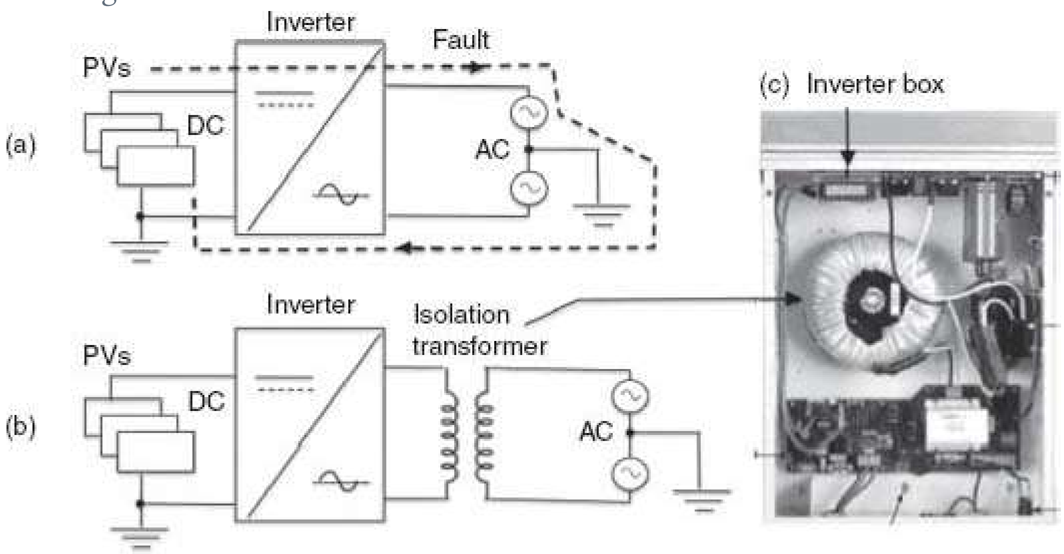
Item	PVWATTS Default	Range
PV module nameplate DC rating	0.95	0.80–1.05
Inverter and Transformer	0.92	0.88–0.98
Module mismatch	0.98	0.97–0.995
Diodes and connections	1.00	0.99–0.997
DC wiring	0.98	0.97–0.99
AC wiring	0.99	0.98–0.993
Soiling	0.95	0.30–0.995
System availability	0.98	0.00–0.995
Shading	1.00	0.00–0.995
Sun tracking	1.00	0.95–1.00
Age	1.00	0.70–1.00
Total derate factor without NOCT	0.770	

A few clarifications to Table 6.2 are in order. The PV module nameplate DC rating refers to the fact that modules coming off the assembly line may all have the same manufacturer nameplate rating, but not all of them may produce that much power even under standard test conditions in the field. A separate but related entry, designated as "Age, " allows users to try to account for the long-term decrease in module efficiency. Studies suggest degradation rates on the order of 0.5%/yr may be likely for crystal silicon (c-Si.)

Newer thin-film modules have improved and are now expected to age at a similar rate to c-Si (Jordan et al., 2011; Marion et al., 2005).

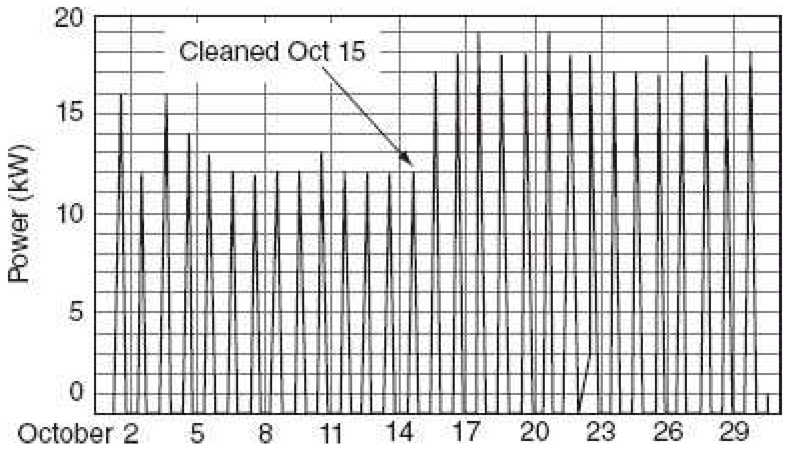
Inverter efficiency is usually very high as long as it operates with relatively high load factors. For safety, most systems include an isolation transformer to prevent fault currents from the DC side of the system to be passed onto the AC grid (Fig. 6.7). These transformers not only contribute to losses, but also are expensive and take up a fair amount of room inside the inverter box. Transformers can be avoided by not grounding the DC portion of the system, which is common in Europe, but is still under consideration in the United States.

FIGURE 6.7 Isolation transformers prevent fault currents from passing from the DC side of the system onto the AC grid connection (a) Showing fault; (b) the isolation transformer; (c) illustrating the size of the transformer.



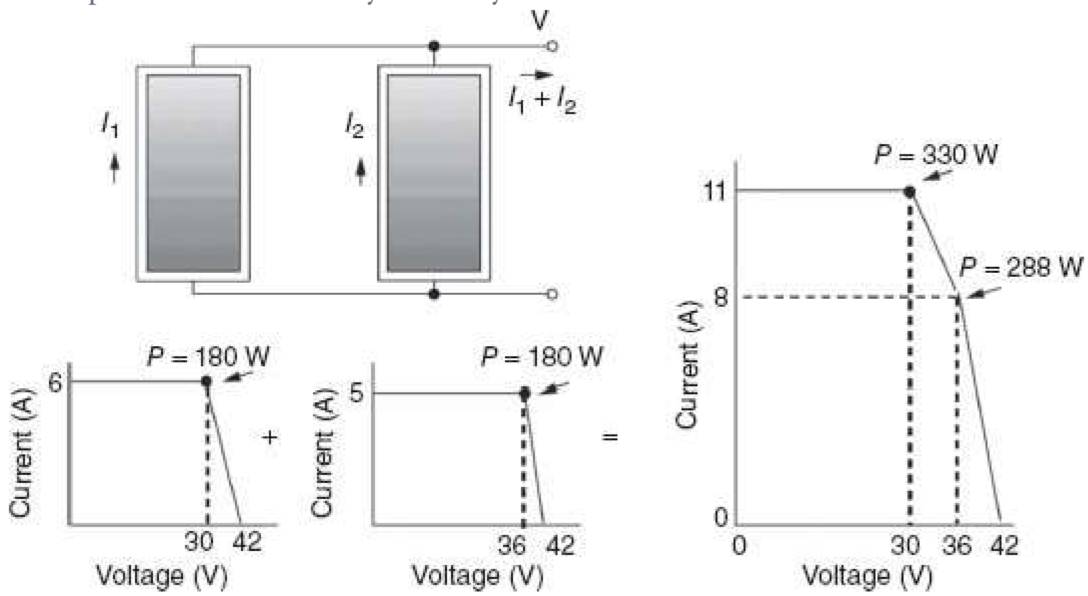
The soiling derate is highly variable depending on rainfall, collector tilt, sources of soiling (including snow), and whether or not periodic washing occurs. Figure 6.8 shows an example of a washing experiment that was conducted in early fall on a nearly horizontal array located in an open field on the edge of the Stanford campus. California summers are characterized by having virtually zero rainfall and the array had never been washed before. After the long summer, the 20-kW array was producing about 12 kW at midday. After washing, the peak output increased by almost 50%.

FIGURE 6.8 A washing experiment conducted in early fall 2008 on a 20-kW nearly horizontal array on the edge of the Stanford campus.



Module mismatch accounts for slight differences in *I*–*V* curves even though modules may deliver the same power under standard test conditions. For example, Figure 6.9 shows two mismatched 180-W modules wired in parallel. They each have the same rated output, but their somewhat idealized *I*–*V* curves have been drawn so that one produces 180 W at 30 V and the other does so at 36 V. As shown, the sum of their *I*–*V* curves shows the maximum power of the combined modules is only 330 W instead of the 360 W that would be expected if their *I*–*V* curves were identical. Note that if microinverters were used, there would be no module mismatch for this example.

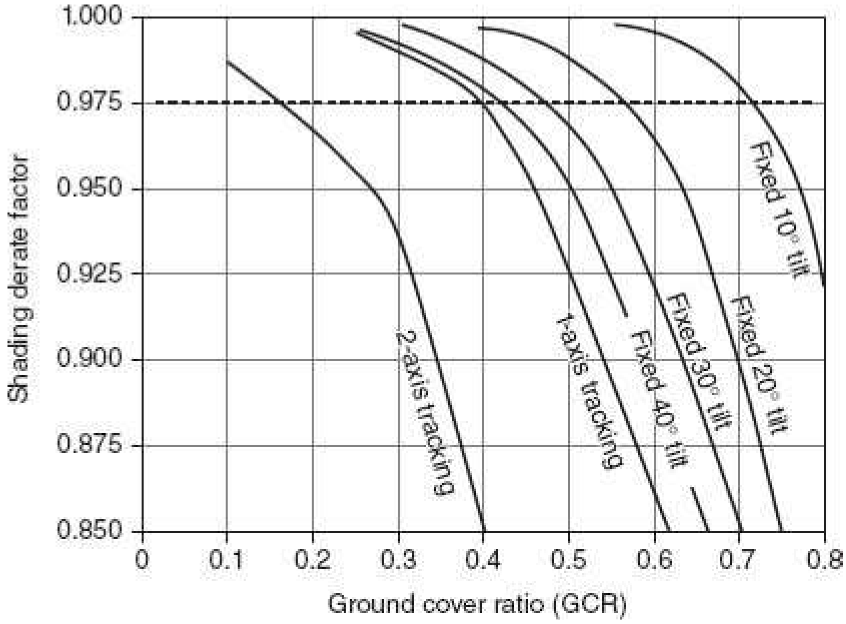
FIGURE 6.9 Illustrating the loss due to mismatched modules. Each module is rated at 180 W but the parallel combination yields only 330 W at MPP.



Shading losses can be the result of nearby obstructions or debris on the panels, but it can also be caused by one collector shading another during certain parts of the day. When there are area constraints, which is often the case on buildings in general, and especially if the roof is flat, which is the case on many commercial buildings, a design decision must be made between horizontal collectors that can fill the space with no shading versus rows of tilted collectors, which may cause shading from one row to another. The tilted collector option can use fewer collectors to get the job done and stacking them in the rows eliminates future access problems if modules need attention.

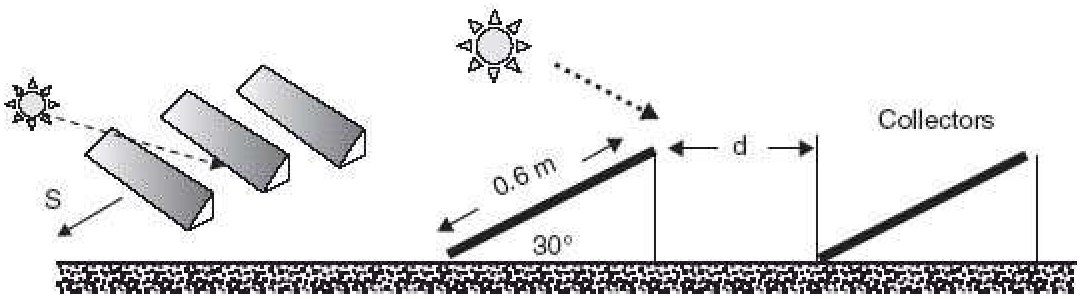
In Section 4.6, shadow diagrams were used to help predict shading problems. A different approach is provided by Figure 6.10, in which shading derate factors for various array configurations are plotted versus a quantity called the ground cover ratio (GCR). The GCR is the ratio of the area of the PVs themselves to the total ground area. Smaller GCR means collectors are spaced further apart so shading is reduced and the derate factor is improved. The PVWATTS website from which this figure was taken indicates that industry practice is to optimize the use of space by configuring the PV system for a GCR that corresponds to a shading derate factor of 0.975 (2.5% loss). Example 6.1 illustrates its use.

FIGURE 6.10 Shading derate factors for various collector configurations. The dashed line suggests an optimum derate of 0.975 (from PVWATTS website).



Example 6.1 Optimum Spacing of Rows of PVs. The CdTe modules described in Table 5.3 have dimensions of 1.2 × 0.6 m. Using Figure 6.10 and the recommended 0.975 derate factor, what should be the collector spacing between rows if the 1.2-m dimension is on the bottom and these south-facing collectors have a 30° tilt angle.

Solution. Begin with a sketch of the geometry.



From Figure 6.10, it looks like GCR should be about 0.47 to give us a derate factor of 0.975. Just using a per-unit of distance along the collector row and the definition of the GCR gives

$$GCR = \frac{\text{Collector area } A_C}{\text{Total ground area } A_{Tot}} = \frac{0.6 \times 1}{(0.6 \cos 30° + d) \times 1} = 0.47$$

$$0.47 \times 0.6 \cos 30° + 0.47d = 0.6$$

$$\text{Spacing } d = \frac{0.6 - 0.2442}{0.47} = 0.76 \text{ m}$$

6.3.2 Temperature-Related PV Derating

Note that PVWATTS derate estimates do not include the very important loss due to cells operating at temperatures above the STC reference temperature of 25°C. That is because PVWATTS uses location-specific typical meteorological year (TMY), hour-by-hour estimates of insolation and ambient temperature to account for losses due to elevated cell temperatures. Recall from Section 4.13.1, how TMY data can be used to find hourly insolation on modules. Hourly cell temperature can be estimated from that insolation along with TMY ambient temperatures and the normal operating cell temperature (NOCT) provided by manufacturers (Section 5.7). NOCT, which is based on an assumed irradiation S of 0.8 kW/m^2 and an assumed ambient temperature of 20°C, can be adjusted for actual conditions using Equation 5.23

$$T_{cell} = T_{amb} + \left(\frac{\text{NOCT} - 20°\text{C}}{0.8 \text{ kW/m}^2}\right) \cdot S(\text{kW/m}^2) \quad (6.2)$$

Using cell temperature estimates calculated from TMY data and Equation 6.2, coupled with the module's temperature coefficient of P_{max}, it is easy to compute hourly NOCT derate factors as the following example illustrates.

Example 6.2 Using TMY Data to Estimate Hourly NOCT Derate Factors. In Example 4.13, calculations based on Atlanta TMY data indicated that 753 W/m^2 would strike a collector at 52° tilt angle, 20° azimuth angle on May 21 at noon. At that time, TMY ambient temperature is 25.6°C. Find the temperature derate factor at noon for the single-crystal silicon (sc-Si) module in Table 5.3 (NOCT = 45°C, temperature coefficient of P_{max} = −0.38%/°C).

Solution. From (6.2)

$$T_{cell} = T_{amb} + \left(\frac{\text{NOCT} - 20°\text{C}}{0.8 \text{ kW/m}^2}\right) \cdot S(\text{kW/m}^2)$$

$$T_{cell} = 25.6 + \left(\frac{45 - 20}{0.8}\right) \cdot 0.753 = 49.1°\text{C}$$

Since the temperature coefficient is based on a comparison with the STC cell temperature of 25°C, the degradation of P_{max} will be

$$\text{Decrease in } P_{max} = 0.38\%/°\text{C} \cdot (49.1 - 25°\text{C}) = 9.16\%$$

That translates to a noon temperature derate factor of

$$\text{NOCT derate} = 1 - 0.0916 = 0.908$$

That is, a 9.2% decrease in performance.

Table 6.3 extends the calculations shown in Example 6.2 to show a full day's worth of temperature derate factors. Notice a few hours in the morning, with cool temperatures and not much insolation, have cells below 25°C; so during those hours, the cells perform better than they would under STCs (i.e., derates are above 1.0).

TABLE 6.3 Hourly NOCT Derate Factors for Atlanta, May 21, Example 6.2

TMY Time	Insolation I_C (kW/m^2)	T_{amb} (°C)	T_{cell} (°C)	NOCT Derate	Effective I_C (kW/m^2)
5.00		16.1	16.1	1.034	
6.00	0.011	16.1	16.4	1.033	0.011
7.00	0.162	16.7	21.8	1.012	0.164
8.00	0.436	18.3	31.9	0.974	0.425
9.00	0.437	20.0	33.7	0.967	0.423
10.00	0.545	21.7	38.7	0.948	0.516
11.00	0.826	23.3	49.1	0.908	0.750
12.00	0.753	25.6	49.1	0.908	0.684
13.00	0.669	26.1	47.0	0.916	0.613
14.00	0.568	25.6	43.4	0.930	0.529
15.00	0.495	26.1	41.6	0.937	0.464
16.00	0.259	26.7	34.8	0.963	0.249
17.00	0.168	26.7	31.9	0.974	0.163
18.00	0.101	25.6	28.8	0.986	0.100
19.00	0.051	23.9	25.5	0.998	0.051
Total	5.480				5.141
		Daily NOCT derate factor = 0.938			

The question arises as to how to find a total daylong derate. Since the point of derate factors is to account for reductions in energy delivered caused by factors such as temperature, it makes sense to care more about derates when insolation is high since a given derate will cause more energy to be lost during those hours. The final column in Table 6.3 is the product of that hour's derate times the insolation during that hour. By totaling the raw insolation column and the effective insolation column after the derates have been applied, we get before and after total kWh/m^2 of insolation. The ratio of the two, in this case 0.938, is the overall daylong temperature derate factor. Over a day's time, we have lost the equivalent of 6.2% of the insolation, which translates directly into 6.2% loss in energy delivered by the PVs.

The example shown in Table 6.3 was derived for a day with an average daytime temperature of 23°C. Cell temperature effects on this pleasant spring day will give a derating factor of 0.938, which means it will cut PV production by 6.2%. In the winter, when it is colder, the temperature derate impact will be less; in the summer, it will more. This 0.938 might, in fact, be a pretty good estimate for the entire year, but to confirm that would require a full 365-day analysis. We will leave that to solar calculators such as PVWATTS (in fact, working backward through PVWATTS suggests their overall, yearlong temperature derate for Atlanta is 0.920, which is an 8.0% loss).

There is another very useful solar calculator online; this one is from the California Energy Commission (CEC). The CEC calculator supports the California Solar Initiative (CSI) by helping consumers figure out the PV incentives offered by the three major investor-owned utilities in California. The CSI approach begins by adjusting the DC, STC rated power of modules to predict their output under what are called PVUSA test conditions (PTC), which are defined as 1-sun irradiance in the plane of the array, 20°C ambient temperature and a windspeed of 1 m/s. The financial incentives are based on module DC, PTC ratings, inverter efficiency, and the location and orientation of the array. Since users enter the actual modules and inverters being considered, the CSI calculator provides direct comparisons of the predicted performance of the major system components.

6.3.3 The “Peak-Hours” Approach to Estimate PV Performance

Predicting performance is a matter of combining the characteristics of the major components—the PV array and the power conditioning unit—with local insolation and temperature data. After having adjusted DC power under STC to expected AC from the PCU using an appropriate derate factor, the next key thing to evaluate is the amount of sunlight available at the site. Chapter 4 was devoted to developing equations for clear-sky insolation, and tables of values are given in Appendix D and E for clear skies. In Appendix G, there are tables of estimated average insolations for a number of locations in the United States.

If the units for daily, monthly, or annual average insolation are specifically kWh/m^2/d, then there is a very convenient way to interpret that number. Since 1-sun of insolation is defined as 1 kW/m^2, we can think of an insolation of say 5.6 kWh/m^2/d as being the same as 5.6 h/d of 1-sun or 5.6 h of “peak sun.” So, if we know the AC power delivered by an array under 1-sun insolation (P_{AC}), we can simply multiply that rated power by the number of hours of peak sun to get daily kWh delivered.

To see whether this simple approach is reasonable, consider the following analysis. We can write the energy delivered in a day's time as

$$\text{Energy (kWh/d)} = \text{Insolation}\left(\frac{\text{kWh/m}^2}{\text{day}}\right) \cdot A(\text{m}^2) \times \bar{\eta} \qquad (6.3)$$

where A is the area of the PV array and $\bar{\eta}$ is the average system efficiency over the day.

When exposed to 1-sun of insolation, we can write for AC power from the system

$$P_{AC}(\text{kW}) = \left(\frac{1\ \text{kW}}{\text{m}^2}\right) \cdot A(\text{m}^2) \times \eta_{1\text{-sun}} \qquad (6.4)$$

where $\eta_{1\text{-sun}}$ is the system efficiency at 1-sun. Combining Equations 6.3 and 6.4 gives

$$\text{Energy (kWh/d)} = P_{AC}(\text{kW}) \left[\frac{\text{Insolation (kWh/m}^2\text{/d)}}{1\ \text{kW/m}^2}\right] \cdot \left(\frac{\bar{\eta}}{\eta_{1\text{-sun}}}\right) \qquad (6.5)$$

If we assume that the average efficiency of the system over a day's time is the same as the efficiency when it is exposed to 1-sun, then the energy collected is what we hoped it would be

(6.6) $$\text{Energy (kWh/d)} = P_{AC}(\text{kW}) \cdot (\text{h/d of "peak sun"})$$

The key assumption in Equation 6.6 is that the system efficiency remains pretty much constant throughout the day. The main justification is that these grid-connected systems have MPPTs that keep the operating point near the knee of the *I–V* curve all day long. Since power at the maximum point is nearly directly proportional to insolation, system efficiency should be reasonably constant. Cell temperature also plays a role, but it is less important. Efficiency might be a bit higher than average in the morning, when it is cooler and there is less insolation, but all that will do is make Equation 6.6 slightly conservative.

Combining Equation 6.1 with Equation 6.6 gives us a way to do simple calculations to estimate annual energy production from a PV array. With the help of PVWATTS, it can also give us a bit more insight into that elusive overall derate factor when it includes temperature effects.

(6.7)

$$\text{Energy (kWh/yr)} = P_{DC,STC} \times \text{Derate factor} \times (\text{h/d of "peak sun"}) \times 365 \text{ d/yr}$$

Typical values for the overall derate factor in the range of 0.70–0.75 seem appropriate, with the lower end of the range applying to hotter climate areas.

Example 6.1 Annual Energy Using the Peak-Sun Approach. A south-facing, 5-kW (DC, STC) array in Atlanta, GA, has a tilt angle equal 18.65° (L–15) for which PVWATTS estimates the annual insolation is 5.12 kWh/m^2/d.

a. Using the peak hours approach estimate the annual energy that will be delivered. This is a relatively warm climate area so let us assume an overall derate factor of 0.72.

b. Then, use the PVWATTS online calculator to compare results. From that result, determine the value of the overall annual derate factor that works in Equation 6.7. With 0.77 being the default nonthermal derate factor, what would be the derate for temperature alone?

c. What improvement would be realized if modules had microinverters that eliminate the module mismatch derate and which also replace the PVWATTS 2% derate DC wiring losses with 1% AC wiring loss?

Solution

a. From Equation 6.7, our estimate is

$$\text{Energy} = 5 \text{ kW} \times 0.72 \times 5.12 \text{ h/d} \times 365 \text{ d/yr} = 6727 \text{ kWh/yr}$$

b. PVWATTS for zip code 30303 using the 0.77 derate factor for everything but temperature, gives 6624 kWh/yr.

The overall derate factor including thermal impacts that PVWATTS must be using is

$$\text{kWh/yr} = P_{\text{DC,STC}} \times \text{Overall derate} \times (\text{h/d peak sun}) \times 365\ \text{d/yr}$$

$$\text{Overall derate} = \frac{6624\ \text{kWh/yr}}{5\ \text{kW} \times 5.12\ \text{h/d} \times 365\ \text{d/yr}} = 0.709$$

Since PVWATTS uses 0.77 as the derate for everything but temperature, that suggests

$$\text{Temperature derate} = \frac{\text{Overall derate}}{\text{Default } 0.77} = \frac{0.709}{0.77} = 0.921 \text{ or about } 7.9\% \text{ loss.}$$

c. Adjusting the defaults in Table 6.2 changes module mismatch from 0.98 to 1.0 and using AC instead of DC wiring changes their loss factor from 0.98 to 0.99. The 0.77 nonthermal derating then becomes

$$\begin{aligned}\text{New derate value} &= 0.95 \times 0.92 \times 1.0 \times 1.0 \times 0.99 \times 0.99 \times 0.95 \times 0.98 \\ &= 0.798\end{aligned}$$

So the total derate with microinverters including our newly found temperature impact is

$$\text{Total derate} = 0.921 \times 0.798 = 0.735$$

So our new peak-hour estimate of annual energy using microinverters is

$$\text{Energy} = 5\ \text{kW} \times 0.735 \times 5.12\ \text{h/d} \times 365 = 6838\ \text{kWh/yr}$$

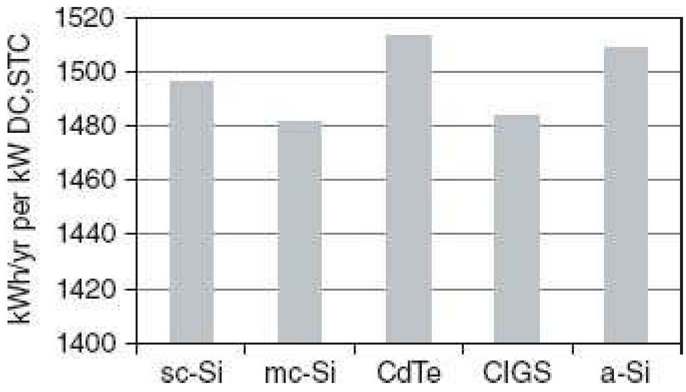
That is a boost of 6838 − 6624 = 214 kWh/yr, about 3.2% over the default PVWATTS estimate.

Some time ago, Scheuermann et al. (2002) measured 19 PV systems in California and found the actual derate between 0.53 and 0.70. On the other hand, early monitoring of NREL's net-zero energy Research Support Facility (RSF) in Golden, CO, resulted in a much better total derate factor of 0.84 (Blair et al., 2012). Over time, there has been a general trend toward more efficient components, better maintenance, and greater care during installation that is showing up in better derate factors.

6.3.4 Normalized Energy Production Estimates

With online calculators readily available, we can easily derive performance data for different PV technologies in different locations. One handy way to make such comparisons is by using manufacturer-provided $P_{\text{DC, STC}}$ ratings to normalize the predicted kWh/yr system outputs. For example, Figure 6.11 shows the normalized outputs of the five Table 5.3 collectors that we have been using for examples. Note, however, that the scale emphasizes what are relatively small differences; between the worst and the best it is only 2%.

FIGURE 6.11 Normalized outputs by PV technology for the modules in Table 5.3 for Palo Alto, CA, tilt angle 38°, 94.5%-efficient inverter, using the CA CSI calculator.



The normalized outputs in Figure 6.11 for the five technologies can be explained quite well by using the temperature deratings under NOCT conditions:

(6.8) $$\text{Power loss} = P_{DC,STC} \cdot \text{Temperature coefficient } (\%/^\circ\text{C}) \cdot (\text{NOCT} - 25)^\circ\text{C}$$

$$\text{sc-Si} = \frac{0.38\%}{^\circ\text{C}} \cdot (45 - 25)^\circ\text{C} = 7.6\%$$

$$\text{mc-Si} = \frac{0.45\%}{^\circ\text{C}} \cdot (46 - 25)^\circ\text{C} = 9.4\%$$

$$\text{CdTe} = \frac{0.25\%}{^\circ\text{C}} \cdot (45 - 25)^\circ\text{C} = 5.0\%$$

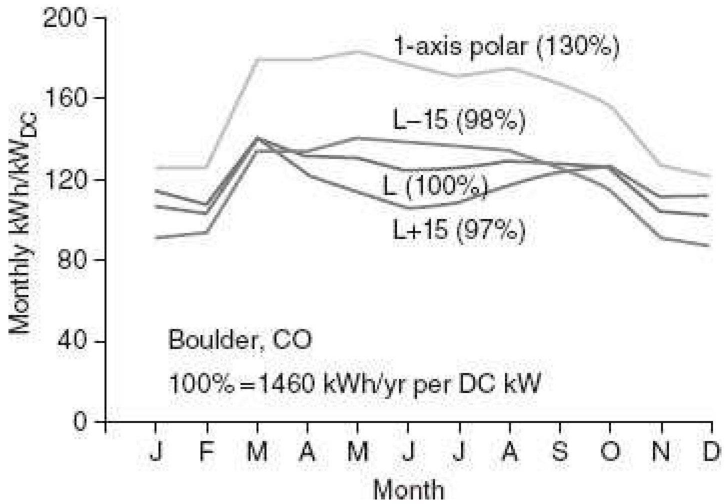
$$\text{CIGS} = \frac{0.40\%}{^\circ\text{C}} \cdot (47 - 25)^\circ\text{C} = 8.8\%$$

$$\text{a-Si} = \frac{0.24\%}{^\circ\text{C}} \cdot (45 - 25)^\circ\text{C} = 4.8\%$$

Note how these temperature impacts alone suggest slightly better (per kW_{DC}) performance might be expected from cadmium telluride (CdTe) and triple-junction amorphous silicon (a-Si) technologies than the others, which seems to be confirmed by the examples shown in Figure 6.11.

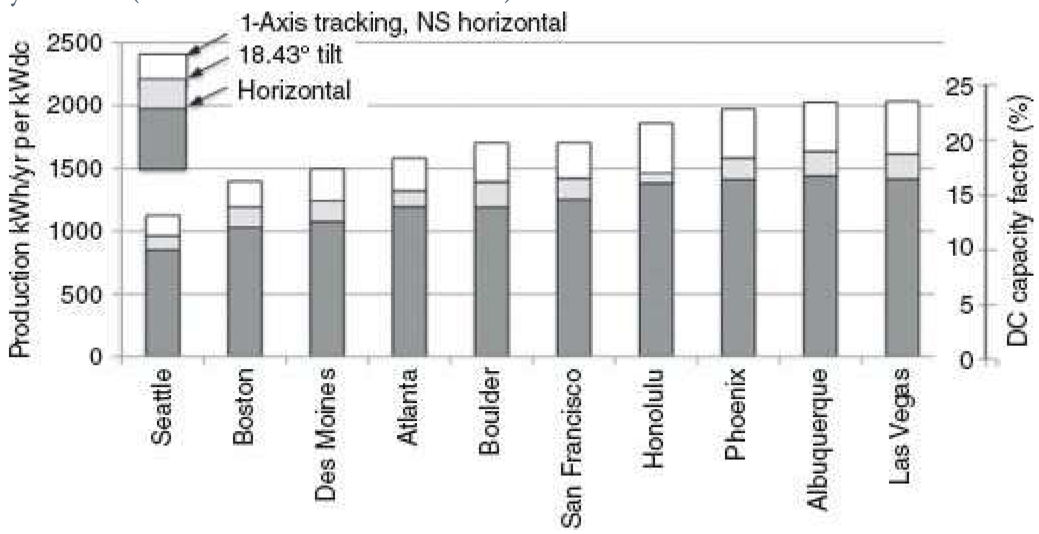
Figure 6.12 presents monthly, normalized energy production for an example location (Boulder, CO) as a way to point out that the tilt angle for net-metered PVs is not particularly important on an annual kWh basis. The difference in performance between an L−15 tilt angle (latitude minus 15°) and L+15 is only a couple of percent. In fact, the reduction at a standard 4-in-12 roof pitch of 18.43° is only 5%. With TOU rates, shallow-pitched, net-metered systems can more than offset those losses by selling more electricity in the summers when it is more valuable. Note, by the way, the sizeable 30% improvement in kWh with single-axis tracking. Two-axis tracking is only 6% better than single-axis tracking.

FIGURE 6.12 Comparing monthly energy production for fixed-tilt arrays and single-axis tracking. The percentages are annual totals relative to a fixed tilt equal to the local latitude (L).



Normalized expected energy productions for a number of U.S. cities are shown in Figure 6.13. The configurations chosen are most representative of residential and commercial installations (horizontal arrays, arrays pitched at a typical 4-in-12 roof slope (18.43°), and horizontal single-axis tracking arrays). For typical roof pitch, south-facing collectors in areas with good, 5.5 full-sun h/d will deliver about 1500 kWh/yr of AC energy per $kW_{DC,\ STC}$.

FIGURE 6.13 Normalized electricity production for a range of U.S. cities. Fixed arrays at typical 4-in-12 roof pitch, horizontal, and 1-axis tracking with horizontal NS axis. Also DC capacity factors (from PVWATTS calculator).



6.3.5 Capacity Factors for PV Grid-Connected Systems

A simple way to present the energy delivered by any electric power generation system is in terms of its rated AC power P_{AC} and its capacity factor (CF). As described in Section 1.6.2, capacity factor over a period of time, usually 1 year, is the ratio of the energy actually delivered to the energy that would have been delivered if the system ran at full rated power all the time. It can also be thought of as the ratio of average power to rated power.

The normal governing equation for annual performance of power plants in terms of CF is based on the annual AC rating of the plant:

(6.9) $$\text{Energy (kWh/yr)} = P_{AC}(\text{kW}) \cdot \text{CF} \cdot 8760\ (\text{h/yr})$$

where 8760 is the product of 24 h/d × 365 d/yr. Monthly or daily capacity factors are similarly defined.

For PVs, it is a common practice to describe performance in terms of their DC capacity factor

(6.10) $$\text{Energy (kWh/yr)} = P_{DC,STC}(\text{kW}) \cdot \text{CF}_{DC} \cdot 8760\ (\text{h/yr})$$

Combining this with the "peak-hours" approach to delivered energy

(6.11) $$\text{Energy (kWh/yr)} = P_{DC}(\text{kW}) \cdot (\text{Derate}) \cdot (\text{h/d full sun}) \cdot 365\ \text{days/yr}$$

leads to the following description of a DC capacity factor

(6.12) $$\text{DC capacity factor (CF}_{DC}) = \frac{\text{h/d of "peak sun"}}{24\ \text{h/d}} \times \text{Derate factor}$$

For normalized PV outputs with units of kWh/yr per $KW_{DC,\ STC}$ (Eq. 6.10) leads to another way to express DC capacity factors:

(6.13) $$\text{CF}_{DC} = \frac{(\text{kWh/yr})/\text{kW}_{DC,STC}}{8760\ \text{h/yr}}$$

Figure 6.13 shows the DC capacity factors for those selected cities.

6.3.6 Some Practical Design Considerations

With the utility there to provide energy storage and backup power, sizing grid-connected systems is not nearly as critical as it is for stand-alone systems. Sizing can be more a matter of how much unshaded area is conveniently available on the building coupled with the customer's budget. Oversizing with net-metered systems can be an issue since the utility may not be willing to pay for any excess energy.

With solar calculators such as PVWATTS and the CSI readily available online, sizing the number of modules needed to meet design goals is quite straightforward. For a paper-and-pencil analysis, or preparing your own spreadsheet to play with, sizing can begin with establishing a kWh/yr target, followed by a simple calculation of the peak watts of DC PV power needed and the collector area required. Or it can start with the area available, from which you can work backward to determine the kWh/yr that can be produced and the peak watts that will make it happen.

At this point, the design begins to be determined by the modules and inverter chosen. Unless the chosen modules have their own microinverters, the array will consist of parallel strings of modules, with the number of modules in a string determined by maximum voltages allowed by code as well as the input voltages needed by the inverter.

Example 6.4 System Sizing in Silicon Valley, CA. Size a PV system to supply 5000 kWh/yr to a home in Silicon Valley. Do the calculations by hand-making assumptions as needed. Test the final design using the CSI calculator (Zip code 94305).

Solution. Assume the roof is south facing with a 4-in-12 pitch (18.43° tilt). PVWATTS estimates 5.32 kWh/m^2/d of insolation. It is a relatively cool region, so let us assume a derate factor of 0.75. Then, using Equation 6.11

$$P_{DC}(\text{kW}) = \frac{\text{Energy (kWh/yr)}}{(\text{Derate}) \cdot (\text{h/d full sun}) \cdot 365\ \text{day/yr}}$$

$$= \frac{5000\ \text{kWh/yr}}{0.75 \times 5.32\ \text{h/d} \times 365\ \text{d/yr}} = 3.43\ \text{kW}$$

Let us assume top quality c-Si modules with 19% efficiency. Using STCs, the PVs need to deliver 3.43 kW, so the area required can be found from

$$P_{DC,STC} = 1\ \text{kW/m}^2 \times A\ (\text{m}^2) \times \eta$$

$$A = \frac{3.43\ \text{kW}}{1\ \text{kW/m}^2 \times 0.19} = 18.05\ \text{m}^2$$

Having confirmed that there is sufficient area on the roof, suppose we pick a SunPower 240-W module with the following key STC characteristics:

Peak power	240 W
Rated voltage V_{MPP}	40.5 V
Open-circuit voltage V_{OC}	48.6 V
Short-circuit current I_{SC}	6.3 A
Temperature coefficient of power	−0.38%/K
Temperature coefficient of V_{OC}	−0.27%/K
Temperature coefficient of I_{SC}	−0.05%/K
NOCT	45°C
Dimensions	1.56 m × 0.798 m = 1.245 m^2/module

We are going to need about 3.43 kW/0.240 kW = 14.3 modules. Before we decide on 14 or 15 modules, let us consider the number of modules per string. To do that, we need an example inverter. Let us try the SunPower 5000 with the following important characteristics:

Maximum power	5000 W
MPP tracking voltage range	250–480 V
Range of input operating voltage	250–600 V
PV start voltage	300 V
Maximum DC input current	21 A
Maximum input short circuit current	36 A

Start with the inverter MPP tracking voltage range of 250–480 V. At 40.5 V per module that suggests a range of 250 V/40.5 V = 6.2 to 480 V/40.5 V = 11.9 modules per string.

We need to see how that changes with temperature. Suppose the coldest daytime temperature that might be expected is −5°C, which is 30° colder than the 25° STC temperature. When it is cold, voltage increases, so at that cell temperature we might expect the MPP voltage to be

$$V_{MPP} = 40.5\text{ V} \cdot [1 - 0.0027(-5 - 25)] = 43.8\text{ V}$$

which means we need fewer than 480 V/43.8 V = 10.9 modules per string to stay in the MPP bounds.

The National Electrical Code restricts all voltages on residential systems in one- and two-family dwellings to no more than 600 V, which is also the inverter limit, so we need to check that constraint. On the coldest day, the maximum module V_{OC} will be

$$V_{OC} = 48.6\text{ V} \cdot [1 - 0.0027(-5 - 25)] = 52.5\text{ V}$$

which tells us we need fewer than 600 V / 52.5 V = 11.4 modules per string. So, the two cold weather tests suggest no more than 10 modules per string.

Now we need to test conditions when it is as hot as might be expected. Assuming the hottest day is 40°C with insolation 1 kW/m^2/d, then the highest cell temperature and MPP voltage will be

$$T_{cell} = T_{amb} + \left(\frac{\text{NOCT} - 20}{0.8}\right) \cdot S = 40 + \left(\frac{45 - 20}{0.8}\right) \cdot 1 = 71.3°\text{C}$$

$$V_{MPP}(\text{hot}) = 40.5\text{ V} \cdot [1 - 0.0027(71.3 - 25)] = 35.4\text{ V}$$

With only 35.4 V and the need to have at least 250 V for the inverter MPPT, that says we need at least 250 V/35.4 V = 7 modules per string.

So, the conclusion is something between 7 and 10 modules per string will satisfy the inverter constraints. Since we decided we needed about 14.3 modules to meet the load, we could be pretty close to our goal by using two strings of seven modules each. With only two strings, each having short circuit current of 6.3 A, means we are well below 31 A maximum current to this inverter.

When this 14 module (3.36 kW_{DC}) system was tested on the CSI solar calculator, their result was 4942 kWh/yr. Using our simple peak-hours approach with a 0.75 derate factor would have predicted the system would deliver

$$14 \times 0.24\text{ kW} \times 5.32\text{ h/d} \times 365\text{ d/yr} \times 0.75 = 4893\text{ kWh/d}$$

which is very close to the result found using the CSI calculator.

6.4 PV SYSTEM ECONOMICS

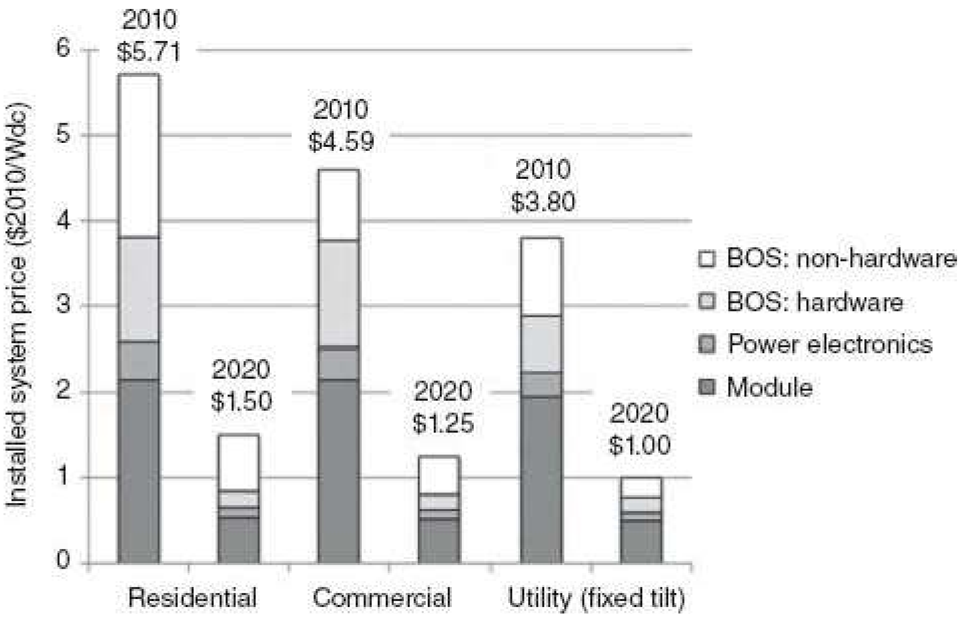
We now have the tools to allow us to estimate the energy delivered by grid-connected PV systems, so the next step is to explore their economic viability. System sizing is pretty similar for both residential and commercial buildings, but their economics differ for several reasons, including some economies-of-scale advantages for larger systems, but more importantly, differences in the economic incentives to each, and differences in the value of the utility power being displaced. Utility-scale systems competing against wholesale power costs are different still, and will be covered in a later section. See Appendix A for a review of some of the basic economic concepts that will be utilized in this section.

6.4.1 PV System Costs

The most important inputs to any economic analysis of a PV system are the initial cost of the system and the amount of energy it will deliver each year. Whether the system is economically viable depends on other factors—most especially, the price of the energy displaced by the system, whether there are any tax credits or other economic incentives, and how the system is to be paid for. A detailed economic analysis will include estimates of operation and maintenance costs, future costs of utility electricity, loan terms and income tax implications if the money is to be borrowed or personal discount rates if the owner purchases it outright, system lifetime, costs or residual value when the system is ultimately removed, and so forth.

Begin with the installed cost of the system. For individual buyers, it is total dollars for their system that matters, but when an overall snapshot of the industry is being presented it is a common practice to describe installed costs in dollars per watt of DC peak power. Figure 6.14 gives us some reference points to help calibrate ourselves to system prices in the recent past and goals for the near future. The 2020 targets shown are those called for in the U.S. Department of Energy's SunShot program, which calls for an overall reduction in the cost of PV power to the point where it can directly compete with incumbent electricity technologies without subsidies.

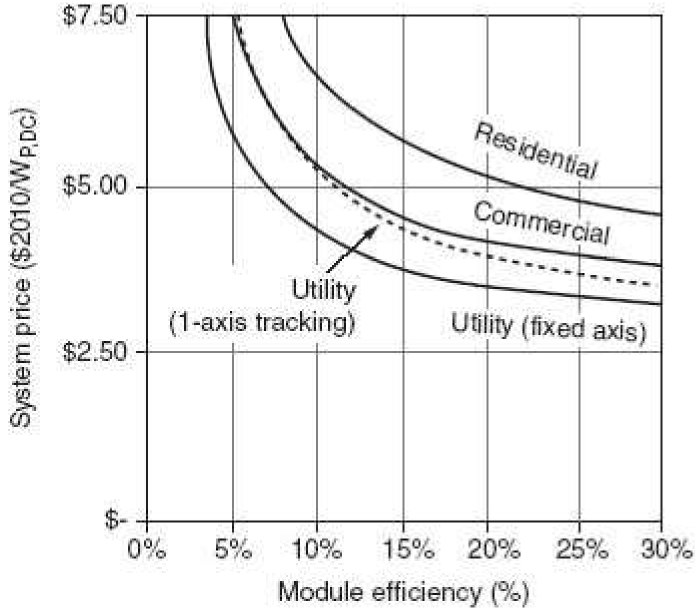
FIGURE 6.14 Estimated 2010 prices for residential and commercial PV systems before incentives, along with SunShot goals for 2020. Redrawn from Goodrich et al. (2012) (NREL).



As shown in Figure 6.14, the 2010 estimated total installed selling price for residential systems in the United States in 2010 was $5.71 per peak DC watt. That was the cost to consumers before any tax incentives or utility rebates were applied. Broken down into categories, 38% was for the modules, 8% for power electronics (mostly the inverter), 22% for noninverter wiring and mounting hardware, and 33% was for the nonhardware balance of systems (BOS). This latter category includes permits, labor, overhead, and profit. The 2020 target is a 75% reduction to $1.50/Wp. Note the commercial prices are modestly lower ($4.59 in 2010 and $1.25 in 2020) with most of that advantage being gained on the nonhardware BOS costs that go with economies of scale. The utility-scale systems in Figure 6.14 are for nontracking, fixed-tilt arrays. Note how it is the reduced cost of the hardware BOS that accounts for much of their 2010 advantage over commercial systems.

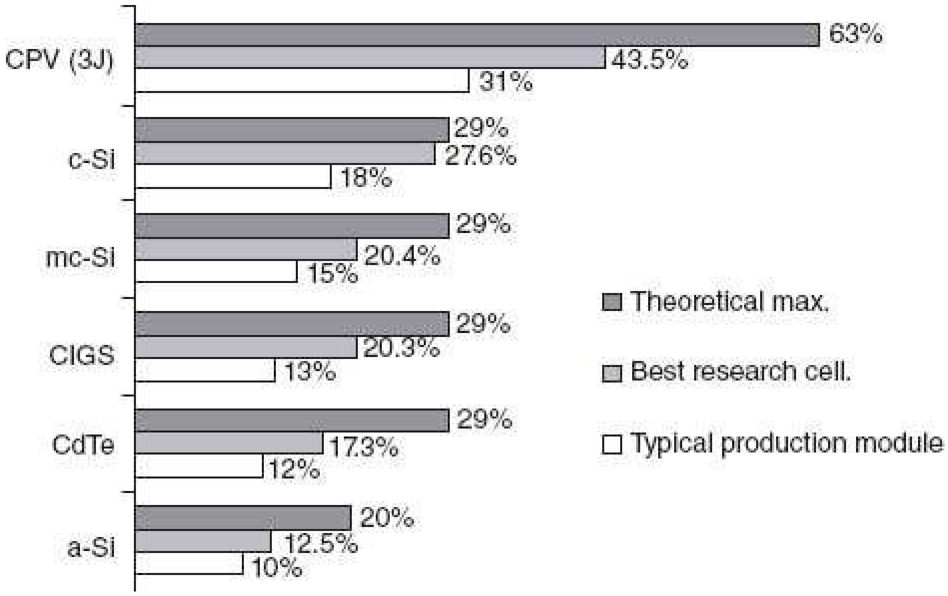
An interesting question arises with PVs located on buildings rather than in open fields where utility-scale systems are installed. When there are area constraints, module efficiency takes on additional importance since the above analysis indicates that roughly two-thirds of system prices are relatively fixed and must be paid for with the kilowatt-hours generated by the modules. In these circumstances, the additional cost of high-efficiency modules can usually be justified by the extra energy they generate. Figure 6.15 shows an NREL sensitivity analysis of the impact of module efficiency on system prices assuming equal $/Wp PV cost.

FIGURE 6.15 Sensitivity of installed PV system price to module efficiency (with equal $/Wp module prices). Redrawn from Goodrich et al. (2012) (NREL).



Significant decreases in module cost and increases in module efficiency are important parts of the SunShot scenario for 2020. Figure 6.16 summarizes the current efficiency status of competing PV technologies along with their theoretical maximum possible efficiencies. sc-Si is closer to its ultimate limit than the others, which suggests there is no much more room for improvement there. It seems likely most future system price reductions will have to come from the nonhardware BOS costs.

FIGURE 6.16 Production, laboratory, and theoretical maximum PV module efficiencies (from NREL, 2012 SunShot Update).



6.4.2 Amortizing Costs

A simple way to estimate the cost of electricity generated by a PV system is to imagine taking out a loan to pay for the system and then using annual payments divided by annual kWh delivered to give $/kWh. If an amount of money, or principal, *P* ($) is borrowed over a period of *n* (years) at an interest rate of *i* (decimal fraction/year), then the annual loan payments, *A* ($/yr) will be

(6.14) $$A = P \cdot \mathrm{CRF}(i, n)$$

where CRF(*i, n*) is the capital recovery factor given by

(6.15) $$\mathrm{CRF}(i, n) = \frac{i(1+i)^n}{(1+i)^n - 1}$$

A short table of capital recovery factors is provided in Table 6.4. These are the conventional per-year values

TABLE 6.4 Capital Recovery Factors

Term	2%	3%	4%	5%	6%	7%	8%	9%	10%
5	0.2122	0.2184	0.2246	0.2310	0.2374	0.2439	0.2505	0.2571	0.2638
10	0.1113	0.1172	0.1233	0.1295	0.1359	0.1424	0.1490	0.1558	0.1627
15	0.0778	0.0838	0.0899	0.0963	0.1030	0.1098	0.1168	0.1241	0.1315
20	0.0612	0.0672	0.0736	0.0802	0.0872	0.0944	0.1019	0.1095	0.1175
25	0.0512	0.0574	0.0640	0.0710	0.0782	0.0858	0.0937	0.1018	0.1102
30	0.0446	0.0510	0.0578	0.0651	0.0726	0.0806	0.0888	0.0973	0.1061

Equations 6.14 and 6.15 were written as if the loan payments are made only once each year. They are easily adjusted to find monthly payments by dividing the annual interest rate *i* by 12 and multiplying the loan term *n* by 12 leading to the following:

$$\text{CRF (monthly)} = \frac{(i/12)[1+(i/12)]^{12n}}{[1+(i/12)]^{12n}-1} \tag{6.16}$$

Example 6.5 Cost of PV Electricity for the Silicon Valley House. The 3.36 kW_{DC} PV system designed in Example 6.4 delivers 4942 kWh/yr. Suppose the system cost is the 2010 residential average of \$5.71/W_{DC} (without incentives). If the system is paid for with a 4.5%, 30-year loan, what would be its cost of electricity? If the SunShot goal of \$1.50/W is achieved, what would the cost be?

Solution. The system will cost \$5.71/W × 3360 W = \$19, 186.

The capital recovery factor (CRF) for this loan would be

$$\text{CRF}(i,n) = \frac{i(1+i)^n}{(1+i)^n-1} = \frac{0.045(1.045)^{30}}{(1.045)^{30}-1} = 0.06139/\text{yr}$$

So the annual payments would be

$$A = P \cdot \text{CRF}(i,n) = \$19{,}186 \times 0.06139 = \$1177.80/\text{yr}$$

The cost per kWh is therefore

$$\text{Cost of electricity} = \frac{\$1066.42/\text{yr}}{4942\ \text{kWh/yr}} = \$0.238/\text{kWh}$$

At the SunShot goal of \$1.50/W, the cost would be

$$\text{Cost of electricity} = \frac{\$1.50/\text{W} \times 3360\ \text{W} \times 0.06139/\text{yr}}{4942\ \text{kWh/yr}} = \$0.062/\text{kWh}$$

That is considerably below the \$0.116/kWh average 2012 U.S. residential price.

A significant factor that was ignored in the cost calculation of Example 6.5 is the impact of the income tax benefit that goes with a home loan. Interest on such loans is tax deductible, which means a person's gross income is reduced by the loan interest, and it is only the resulting net income that is subject to income taxes. The tax benefit that results depends on the marginal tax bracket (MTB) of the homeowner, which is defined for 2012 federal income tax in Table 6.5. For example, a married couple earning \$120, 000 per year is in the 25% MTB, which means a one-dollar tax deduction reduces their income tax by 25 cents. The value of the tax deduction will be even greater when it reduces the homeowner's state income tax as well. For example, the same taxpayer in California would be in the 32% MTB when both state and federal taxes are considered.

TABLE 6.5 Federal Income Tax Brackets for 2012

	Married Filing Jointly	Single
10% bracket	\$0–\$17, 400	\$0–\$8700
15% bracket	\$17, 400–\$70, 700	\$8700–\$35, 350
25% bracket	\$70, 700–\$142, 700	\$35, 350–\$85, 650

29% bracket	\$142, 700–\$217, 450	\$85, 650–\$178, 650
33% bracket	\$217, 450–\$388, 350	\$178, 650–\$388, 350
35% bracket	Over \$388, 350	Over \$388, 350

During the first years of a long-term loan, almost all of the annual payments will be interest, with very little left to reduce the principal, while the opposite occurs toward the end of the loan. That means the tax benefit of interest payments varies from year to year. For our purposes, we will assume loan payments are made once a year at the end of the year. For example, in the first year, interest is owed on the entire amount borrowed and the tax benefit is

(6.17) $\text{First-year tax benefit} = i \times P \times \text{MTB}$

In addition to the tax benefit of deductible interest, there may be local, state, and federal incentives as well as rebates provided by electric utilities. Since these are so variable and location specific, the only one we will mention here is the long-standing 30% federal renewable energy tax credit. Realize that tax credits and tax deductions are quite different incentives. A tax credit is much more valuable since it reduces the buyer's tax burden by the full amount of the credit while a tax deduction reduces it by the product of the deduction times the MTB.

Example 6.6 Cost of PV Electricity Including Tax Benefits. The 3.36-kW_{DC} PV system for the house in Silicon Valley costs \$19, 186 and delivers 4942 kWh/yr. The homeowner finances the net cost of the system after receiving a 30% federal tax credit using a 4.5%, 30-year loan. If the homeowner is in the 25% MTB, what is the cost of PV electricity in the first year?

Solution. The capital cost of the system after the 30% tax credit is

$$P = \$19{,}186(1 - 0.30) = \$13{,}430$$

The CRF for the loan is still 0.06139/yr, so the loan payments will be

$$A = P \cdot \text{CRF}(i, n) = \$13{,}430 \times 0.06139 = \$824.49/\text{yr}$$

During the first year, the owner has use of \$13, 430 for a full year without yet paying any interest. So at the end of the first year, the interest owed is

$$\text{First-year interest} = 0.045 \times \$13{,}430 = \$604.35$$

After making that first \$824.49 payment, \$604.35 of which is interest, the loan principal is reduced by only \$824.49 – \$604.35 = \$220.14.

The first-year tax savings based on the tax-deductible interest portion of the payment is

$$\text{First-year tax savings} = 0.25 \times \$604.35 = \$151.09$$

The net cost of the PV system in that first year will therefore be

$$\text{First-year cost of PV} = \$824.49 - \$151.09 = \$673.40$$

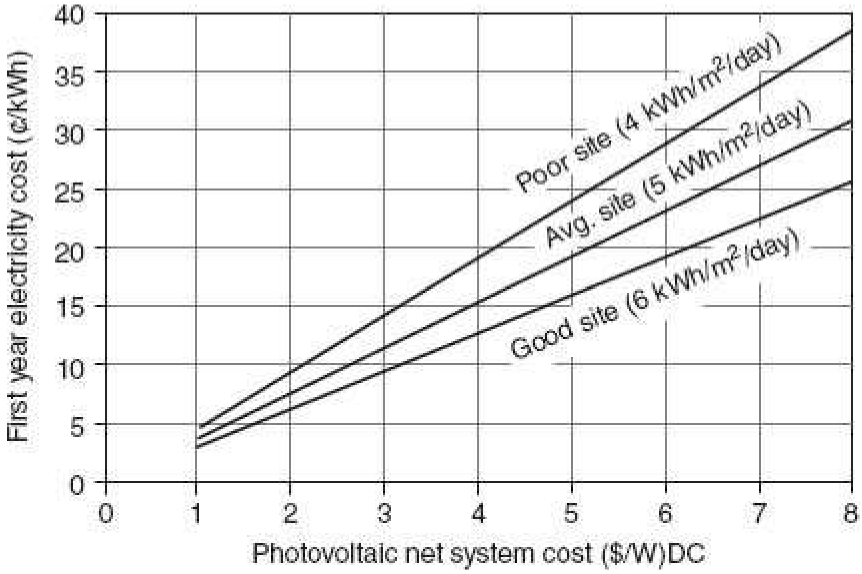
which means the first-year cost of electricity is

$$\text{First-year cost of PV electricity} = \frac{\$673.40/\text{yr}}{4942\ \text{kWh/yr}} = \$0.136/\text{kWh}$$

This is not much more than the 2012 $0.116/kWh average price of electricity in the United States.

Figure 6.17 shows the way the above calculations play out when insolation and capital cost are treated as parameters.

FIGURE 6.17 First-year cost of electricity with net system cost (after rebates) and panel insolation as parameters. Assumptions: 5%, 30-year loan, 25% MTB, 0.75 derate factor.



6.4.3 Cash Flow Analysis

The analysis in Example 6.6 covered only the first year of the PV investment. A straightforward cash flow analysis, however, easily accounts for complicating factors such as PV performance degradation, utility price escalation, declining tax-deductible interest over time, periodic maintenance costs, and disposal or salvage value of the equipment at the end of its lifetime.

Table 6.6 shows portions of a spreadsheet version of a 30-year cash flow analysis for the Silicon Valley PV system described in Examples 6.4–6.6. New inputs include an annual PV performance degradation of 0.5%/yr and a utility price increase of 3%/yr, with both starting at the end of the first year. Note the net cash flow during the first year shows a loss of $49.95 since the net cost of the loan ($623.26) is greater than the utility savings ($573.30). In the 30th year, there is a positive $413.37 cash flow.

TABLE 6.6 Cash Flow Analysis for the Silicon Valley PV System

Rated DC power (kW_{DC})	3.36	ENTER			
Insolation ($kWh/m^2/d =$ h/d)	5.32	ENTER			
Derating	0.7575	ENTER			
First-year production (kWh/yr)	4942				
System degradation (%/yr)	0.5%	ENTER			
Cost before incentives ($/Wp)	$5.71	ENTER			
System cost	$19,185.60				
Rebates (30% default)	$5755.68				
Final cost ($) and ($/Wp)	$13,429.92	$4.00			
Down payment	$1000.00	ENTER			
Loan principal	$12,429.92				
Loan interest (%/yr)	4.5%	ENTER			
Loan Term (yr)	30	ENTER			
CRF (i,n)/yr	0.06139				
Annual payments ($/yr)	$763.09				
Marginal tax bracket (MTB)	25%	ENTER			
Nominal discount rate	5.0%	ENTER			
First-year utility price ($/kWh)	$0.116	ENTER			
Utility	3.0%	ENTER			
Year	0	1	2	. . .	30
Payment ($/yr)	$1000.00	$763.09	$763.09		$763.09
Interest ($/yr)		$559.53	$550.18		$32.86
Delta balance		$203.75	$212.91	. . .	$730.23
Loan balance	$12,429.92	$12,226.17	$12,013.26	. . .	$(0.00)
Tax savings on interest ($/yr)		$139.84	$137.54		$8.22
Net cost of PV		$623.26	$625.55		$754.88
PV kWh generated (kWh/yr)		4942	4918	. . .	4274
Utility price ($/kWh)		$0.116	$0.119		$0.273
Utility cost without PVs		$573.30	$587.55	. . .	$1168.24
Net cash flow ($/yr)	$(1000.00)	$(49.95)	$(38.00)		$413.37
Internal rate of return (IRR)	7.37%				
Net present value (NPV)	$623.45				

There are also two financial measures included in the spreadsheet. One is a cumulative net present value (NPV) calculation and the other is an evaluation of the 30-year internal rate of return (IRR) on the owner's solar investment. Both of these terms are described more carefully in Appendix A, but for now just a simple explanation will be provided. Also, they are both standard functions in Excel so you can easily let your spreadsheet compute these important results.

A present value calculation takes into account the time value of money, that is, the fact that one dollar ten years from now is not as good as having one dollar in your pocket today. A *present worth* or *present value* calculation accounts for this distinction. Imagine having P dollars today that you can invest at an interest rate d. A year from now you would have $P(1 + d)$ dollars in your account; n years from now you would have a future amount of money $F = P(1 + d)^n$ dollars in the account. This means a future amount of money F should be equivalent to having P in our pocket today, where

(6.18) $$P = \frac{F}{(1+d)^n}$$

When converting a future value F into a present worth P the interest term d in Equation 6.18 is referred to as a discount rate. The discount rate can be thought of as the interest rate that could have been earned if the money had been put into the best alternative investment available. For example, in our Table 6.6 spreadsheet, the PV system is projected to save $413.37 in utility bills in the 30th year, so with our discount rate of 5%/yr the present worth of that savings would be

$$P = \frac{F}{(1+d)^n} = \frac{\$413.37}{(1+0.05)^{30}} = \$95.64 \quad (6.19)$$

That is, the savings way out in the 30th year is like putting $95.64 into the owner's pocket today. For our spreadsheet in Table 6.6, the cumulative NPV is a net positive $623.45.

The second financial summary introduced in our spreadsheet is an internal rate of return (IRR). The IRR is perhaps the most persuasive measure of the value of an energy efficiency or renewable energy project since it allows the energy investment to be directly compared with the return that might be obtained for any other competing investment. One simple definition is that the IRR is the discount rate that makes the NPV of the energy investment equal to zero. For our spreadsheet example, in which the owner put in $1000 as a down payment plus a few tens of dollars during the first few years while the system was losing money, the IRR is a respectable 7.37%. That is, the potential buyer of this system would need to find some alternative investment that would earn better than 7.37% to be equivalent to his or her investment in this solar system.

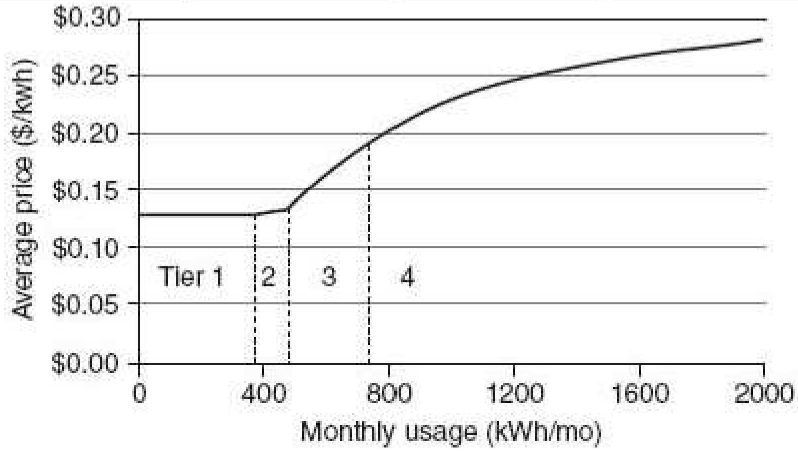
6.4.4 Residential Rate Structures

So far, our analysis has treated utility costs as if they are a simple $/kWh fee along with some future escalation rate. The reality is much more complicated. Electric rates vary considerably, depending not only on the utility itself, but also on the electrical characteristics of the specific customer purchasing the power. The rate structure for a residential customer will typically include a basic fee to cover costs of billing, meters, and other equipment, plus an energy charge based on the number of kilowatt-hours of energy used. Commercial and industrial customers are usually billed not only for energy (kilowatt-hours) but also for the peak amount of power that they use (kilowatts). That *demand charge* for power ($/mo/kW) is the most important difference between the rate structures designed for small customers versus large ones. Large industrial customers may also pay additional fees if their power factor, that is, the phase angle between the voltages supplied and the currents drawn, is outside of certain bounds.

Consider the example residential rate structure shown in Figure 6.18 for one of California's major investor-owned utilities. Note that it includes four tiers based on monthly kWh consumed and also note that the rates increase with increasing demand. This is an example of what is called an *inverted block rate structure*, designed to discourage excessive consumption. Not that long ago, the most common structures were based on *declining block rates*, which made electricity cheaper as the customer's demand increased. The figure shows 365 kWh/mo as the baseline consumption. Actually, that value depends on season and location as well as the home's heating and cooling system. Clearly, this rate structure provides far more financial incentive to invest in a solar system for customers who consume larger amounts of electricity.

FIGURE 6.18 Example of a standard summer residential rate schedule for a California utility. Average consumption for this utility is in Tier 3 (from PG&E E-1, 2012).

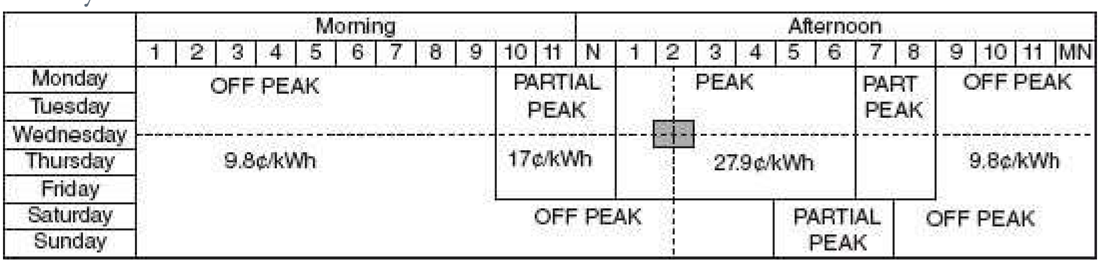
Tier 1 baseline	Tier 2 (101–130% of baseline)	Tier 3 (131–200% of baseline)	Tier 4 (Over 200% of baseline)
e.g. 1st 365 kWh	366–475	475–730	>731 kWh/mo
$0.1285	$0.1460	$0.2956	$0.3356



While the standard rate structure in Figure 6.18 does discourage excessive consumption, it does not address the peak demand issue. A kilowatt-hour used at midnight is priced the same as a kilowatt-hour used in the middle of a hot, summer afternoon when all of the utility's plants may be running at full capacity. In an effort to encourage customers to shift their loads away from peak demand times, many utilities are beginning to offer residential TOU rates.

Figure 6.19 presents an example of the Tier 1 residential summer TOU rate schedule for the same utility shown in Figure 6.18. For comparison, the standard rate is $0.146/kWh, no matter when the energy is used. For the TOU schedule, during off-peak times electricity is only $0.098/kWh, but during peak demands, it is $0.279/kWh. The incentive is certainly there to shift loads whenever possible off of the peak demand period. The other interesting, and tricky, thing to consider is the potential advantages associated with signing up for TOU rates with a net-metered PV system. Selling electricity to the utility at peak rates and buying it back at night-rates can increase the economic viability of PVs. A careful calculation would need to be made to determine whether the TOU rate or the regular residential rate schedule would be most appropriate for an individual homeowner considering PVs.

FIGURE 6.19 An example of time-of-use (TOU) rate schedule. The intersection of the dotted lines suggests how a homeowner might know the rates at 2:00 P.M. on a Wednesday.



With TOU rate structures, prices vary seasonally and by time of day following a predetermined pricing schedule. New dynamic rate structures being introduced take advantage of the ability of smart meters to record electric usage in short time intervals, which creates the potential for time-varying electricity prices. These dynamic rate structures include real-time pricing (RTP) and critical peak pricing (CPP). With RTP, prices vary hourly following the wholesale electricity market conditions. With CPP, customers sign up for a reduced rate for all but a few unscheduled "events." During these CPP events, which are allowed to occur only a limited number of times each year, the price of electricity jumps to a predetermined, very high rate. CPP programs are justified by the cost savings associated with not having to build, and only occasionally operate, peaking power plants.

Table 6.7 shows an example of proposed CPP rate schedule. As designated, CPP events would occur between 4:00 P.M. and 7:00 P.M. on no more than 12 summer days per year, for a total of no more than 36 h. Customers who choose this option will be notified of a CPP event one day in advance. In exchange they receive lower off-peak rates.

TABLE 6.7 A Proposed Critical Peak Pricing Rate Schedule ($/kWh)[a]

Schedule	Off Peak (Base Usage) ($)	Off Peak (Above Base) ($)	Peak ($)	Event ($)
TOU	0.0846	0.1660	0.2700	
TOU + CPP	0.0721	0.1411	0.2700	0.7500

[a]Sacramento Municipal Utility District (SMUD), 2012.

6.4.5 Commercial and Industrial Rate Structures

The rate structures that apply to commercial and industrial customers usually include a monthly demand charge based on the highest amount of power drawn by the facility. That demand charge may be especially severe if the customer's peak corresponds to the time during which the utility has its maximum demand since at those times the utility is running its most expensive peaking power plants.

In the simplest case, the demand charge is based on the peak demand in a given month, usually averaged over a 15-minute period, no matter what time of day it occurs. When TOU rates apply, there may be a combination of demand charges that apply to different time periods as well as the seasons. For the rate structure shown in Table 6.8, one charge applies to the maximum demand no matter what time it is reached and the others are time-period specific. As Example 6.7 points out, these demand charges are additive.

TABLE 6.8 Electricity Rate Structure Including Monthly Demand Charges

Energy ($/kWh)	Off Peak	Partial Peak	Peak
Summer	$0.0698	$0.0950	$0.1336
Winter	$0.0727	$0.0899	
Demand ($/kW/mo)	**Maximum**	**Partial peak**	**Peak**
Summer	$11.85	$3.41	$14.59
Winter	$11.85	$0.21	

Example 6.7 Impact of Demand Charges. During a summer month, a small commercial

building is billed for the following energy and peak demands using the rate structure given in Table 6.8. The maximum peak for the month is 100 kW.

	Off Peak	Partial Peak	Peak
Energy (kWh/mo)	18,000	8000	12,000
Peak demand (kW)	60	80	100

a. Compute the monthly bill.

b. Suppose a 20-kW_{DC} PV system shaves the off-peak, partial-peak, and peak energy loads by 700, 600, and 1400 kWh/mo, respectively. Suppose too, it drops the peak demand by 15 kW during the peak and partial-peak periods. Compute the resulting dollar savings on the utility bill. How much does that work out to per kWh of PV?

Solution

a. Looking at the three time periods, the maximum demand at any time during the month must be 100 kW, which will be charged out at \$11.85/kW = \$1185.

In addition, separate demand charges apply during the peak and partial-peak periods:

$$\text{Peak and partial peak demand} = 80\text{ kW} \times \$3.41 + 100\text{ kW} \times \$14.59 = \$1732$$

$$\text{Total demand charge} = \$1185 + \$1732 = \$2917$$

$$\text{Energy} = 18{,}000 \times \$0.0698 + 8000 \times \$0.0950 + 12{,}000 \times \$0.1336 = \$3620$$

$$\text{Total bill} = \$3619 + \$2917 = \$6536\ (44\%\text{ of which is demand charges})$$

b. With PVs the savings will be

$$\text{Demand savings} = 15\text{ kW} \times (\$11.85 + \$3.41 + \$14.59) = \$443$$

$$\text{Energy \$ savings} = 700 \times \$0.0698 + 600 \times \$0.0950 + 1400 \times \$0.1336 = \$275$$

$$\text{Total savings} = \$443 + \$275 = \$718\ (62\%\text{ is demand savings})$$

Total energy delivered by the PVs = 700 + 600 + 1400 = 2700 kWh

$$\text{PV savings} = \frac{\$718}{2700\text{ kWh}} = \$0.266/\text{kWh}$$

The demand charge in the rate schedule shown in Table 6.8 applies to the peak demand for each particular month in the year. The revenue derived from demand charges for a single month with especially high demand may not be sufficient for the utility to pay for the peaking power plant they had to build to supply that load. To address that problem, some utilities have a ratchet adjustment built into the demand charges. For example, the monthly demand charges may be ratcheted to a level of perhaps 80% of the annual peak demand. That is, if a customer reaches a highest annual peak demand of 1000 kW, then for every month of the year the demand charge will be based on consumption of at least 0.80 × 1000 kW = 800 kW. This can lead to some rather extraordinary penalties for customers who add a few kilowatts to their load right at the time of their annual peak, and conversely, it provides considerable incentive to reduce their highest peak demand.

6.4.6 Economics of Commercial-Building PV Systems

The economics of a behind-the-meter PV system for larger customers differs considerably from that of a residential system. Figure 6.14 showed the capital cost advantage of larger systems and Example 6.7 showed how ignoring demand charges will miss a major attribute of their cost-effectiveness. In addition, there are significant tax advantages that go with these systems.

Businesses are allowed to depreciate their capital investments by writing off the expenditures, which means they get to deduct those costs from their profits before paying corporate taxes. Renewable energy systems can be depreciated using a depreciation scheduled called the Modified Accelerated Cost Recovery System (MACRS). Under MACRS, photovoltaic systems are eligible for the depreciation schedule shown in Table 6.9. If a 30% tax credit is taken, then the amount that can be depreciated is reduced by half of that 30%. The table works out the MACRS financial gain, for an example \$100, 000 PV system. Between the 30% tax credit and the accelerated depreciation, the net effective system cost is reduced by almost 57%.

TABLE 6.9 MACRS Depreciation Schedule

Investment			\$100,000	
30% Investment Tax Credit (ITC)			30,000	
Depreciable basis			85,000	Inv – 50% × ITC
Corporate tax rate			35%	
Corporate discount rate			6%	
Year	**MACRS**	**Depreciation**	**Tax Savings**	**Present Value**
0	20.00%	\$17,000	\$ 5,950	\$ 5,950
1	32.00%	\$27,200	\$ 9,520	\$ 8,981
2	19.20%	\$16,320	\$ 5,712	\$ 5,084
3	11.52%	\$ 9,792	\$ 3,427	\$ 2,878
4	11.52%	\$ 9,792	\$ 3,427	\$ 2,715
5	5.76%	\$ 4,896	\$ 1,714	\$ 1,281
Totals	100%	\$85,000	\$29,750	\$26,887
Effective Net System Cost (Inv – ITC – MACRS):				\$43,113

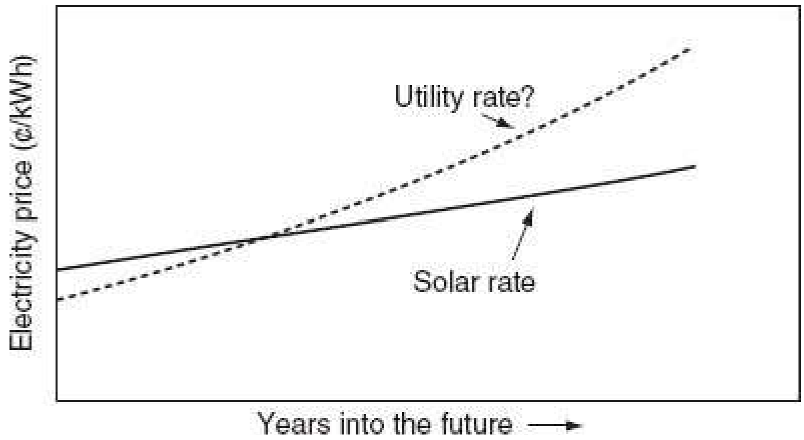
While time-of-use rates with demand charges attempt to capture the true cost of utility service, they are still relatively crude since they only differentiate between relatively large blocks of time (e.g., peak, partial peak, and off peak) and they typically only acknowledge two seasons: summer and nonsummer. The ideal rate structure would be one based on real-time pricing (RTP) in which the true cost of energy is reflected in rates that change throughout the day, each and every day. With RTP, there would be no demand charges, just energy charges that might vary, for example, on an hourly basis.

Some utilities now offer day-ahead, hour-by-hour, real-time pricing for large customers. When a customer knows that tomorrow afternoon the price of electricity will be high, they can implement appropriate measures to respond to that high price. With the price of electricity more accurately reflecting the real, almost instantaneous, cost of power, it is hoped that market forces will encourage the most efficient management of demand.

6.4.7 Power Purchase Agreements

Residential customers cannot depreciate their systems the way businesses can, nonprofit organizations cannot take advantage of tax credits, and many investors do not owe enough taxes to have credits pay off quickly enough. For these and other reasons, many photovoltaic systems are now being installed using third-party financing in which an outside entity contracts to finance, install, and maintain systems on the premises of their customers. In exchange, the customer signs a repayment agreement that specifies the price of electricity generated by the system over the term of the contract. It is not uncommon for these power purchase agreements (PPAs) to start with a slightly higher unit price for solar power than what the utility offers, but the assumption is that over time it will result in net savings for the host (Fig. 6.20).

FIGURE 6.20 An example of PPA in which the initial solar rate is higher than the utility rate.



From the customer's perspective, these PPAs provide a hedge against uncertain future utility price increases and, of course, they help "green" the customer's brand. They require no capital expenditures by the customer and since the host pays only for the kilowatt-hours actually generated, it is the PPA provider who has the incentive to properly operate and maintain the system. From the provider's perspective, they get the tax credits, depreciation allowances, utility rebates, a steady revenue stream, and they may be able to sell the renewable energy credits (RECs) and potential future carbon credits as well.

6.4.8 Utility-Scale PVs

Utility-scale PV systems have economies-of-scale advantages over smaller behind-the-meter systems (Fig. 6.14), but they have the disadvantage of having to compete in the wholesale electricity market where the price is often one-third that of retail rates.

Power purchase agreements are common, but now the customer is a utility rather than a building owner. To account for the variable value of electricity sold during different times of day, many of these PPAs include a generation time-of-delivery (TOD) factor. The original contract includes an agreed upon PPA base rate, which is then adjusted on an hour-by-hour basis using TOD factors such as the ones shown in Table 6.10.

TABLE 6.10 Time-of-Delivery Factors

Season	Period	Definition	Factor
Summer	On Peak	WDxH, noon–6:00 P.M.[a]	3.13
June–	Mid-Peak	WDxH, 8:00 A.M.–noon, 6–11:00 P.M.	1.35
September	Off Peak	All other times	0.75
Winter	Mid-Peak	WDxH, 8:00 A.M.–9:00 P.M.	1.00
October–	Off Peak	WDxH, 6–8:00 A.M., 9:00 P.M.–midnight	0.83
May	Off Peak	WE/H, 6:00 A.M.–midnight[b]	0.83
	Super-Off-Peak	Midnight–6:00 A.M.	0.61

[a]WDxH is defined as weekdays except holidays.
[b]WE/H is defined as weekends and holidays.
(Southern California Edison, 2010).

Example 6.8 Applying TOD Factors for a Utility-Scale PV Plant. A 1000-kW_{DC}, 30° fixed tilt, PV system at 40° latitude has a PPA with a base rate of \$0.10/kWh that is subject to the TOD factors of Table 6.10. Assuming a derate factor of 0.75, find the revenue earned for the noon hour on a clear weekday in June. For the month of June, assume all clear days consisting of 22 weekdays and 8 weekend days; find the average price that will be paid for the energy delivered.

Solution. From the hour-by-hour clear-sky insolation values given in Appendix D, the irradiance at noon on the array will be 960 W/m^2. Assuming that is a good average for the hour, the energy delivered for the noon hour will be

$$\text{Energy (noon)} = P_{DC}(\text{kW}) \cdot \text{Insolation (h full sun)} \cdot \text{Derate}$$

$$= 1000 \text{ kW} \times 0.960 \text{ h} \times 0.75 = 720 \text{ kWh}$$

Noon on a weekday has a TOD of 3.13, so the revenue earned for that hour will be

$$\text{Revenue} = 720 \text{ kWh} \times 3.13 \times \$0.10/\text{kWh} = \$225.36$$

From a spreadsheet, we can work out the rest of the hourly revenues for both weekdays and weekends:

			Weekdays		Weekends	
Solar Time	Insolation (W/m^2)	kWh/d del	TOD X	Revenue $/d	TOD X	Revenue $/d
6	93	70	0.75	$ 5.23	0.75	$ 5.23
7	289	217	0.75	$ 16.26	0.75	$ 16.26
8	498	374	1.35	$ 50.42	0.75	$ 28.01
9	686	515	1.35	$ 69.46	0.75	$ 38.59
10	834	626	1.35	$ 84.44	0.75	$ 46.91
11	928	696	1.35	$ 93.96	0.75	$ 52.20
12	960	720	3.13	$ 225.36	0.75	$ 54.00
1	928	696	3.13	$ 217.85	0.75	$ 52.20
2	834	626	3.13	$ 195.78	0.75	$ 46.91
3	686	515	3.13	$ 161.04	0.75	$ 38.59
4	498	374	3.13	$ 116.91	0.75	$ 28.01
5	289	217	3.13	$ 67.84	0.75	$ 16.26
6	93	70	1.35	$ 9.42	0.75	$ 5.23
Totals		5712		$1313.96		$428.40

Total revenue for a June month with 22 weekdays and 8 weekend days will be

$$\text{Revenue} = 22 \times \$1313.96 + 8 \times \$428.40 = \$32{,}334.32/\text{mo}$$

The energy collected will be 5712 kWh/d × 30 d/mo = 171, 360 kWh/mo.
So the per kWh average payment will be

$$\text{Average rate} = \frac{\$32{,}334.32/\text{mo}}{171{,}360\ \text{kWh/mo}} = \$0.189/\text{kWh}$$

Under the clear-sky assumptions in Example 6.8, a $0.10/kWh PPA actually provides almost double that in revenue for the owner. This illustrates the importance of generating electricity during the times when it is most needed. Since afternoon power is worth more than morning power, the owners will probably orient the array slightly to the west to emphasize that added value. Those TOD factors also point out the economic disadvantage that wind systems have in regions where the strongest winds are at night.

6.5 OFF-GRID PV SYSTEMS WITH BATTERY STORAGE

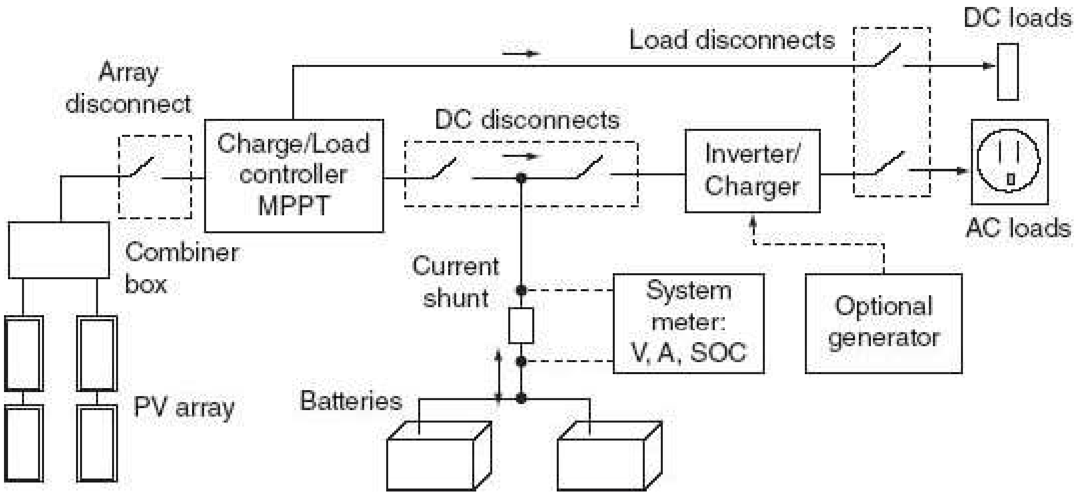
Grid-connected PV systems have a number of desirable attributes. Their relative simplicity can result in high reliability; their MPPT unit assures high PV efficiency; their ability to deliver power to the utility during times of day when it is most valuable increases their economic viability, as does the potential to avoid land costs by installing systems on buildings or parking lots. On the other hand, they have to compete with the relatively low price of utility power, and of course, they are dependent on the grid itself.

When there is no grid available, the competition that PVs face is either the cost of stringing new power lines at tens of thousands of dollars per mile or running noisy, polluting, high maintenance generators burning relatively expensive fuel. For the 1–2 billion people across the globe who currently have little or no access to commercial electricity, having even a little bit of power can transform lives. Even "pico-scale" PV systems with just a few watts of power can replace a kerosene lamp with light-emitting diodes (LEDs), making it possible to read in the evening. A few more watts might let an entrepreneur start a cell phone charging business. Another couple of modules enable lights, a TV, computer, and maybe a few small appliances to be powered. A couple of kilowatts can power a small, modern cabin in the woods and 10 kW will power a room full of computers in a village school as well as a pump for its water supply. String some wires from a modest array on a community center roof and a number of dwellings can be powered with a solar-powered microgrid. Small off-grid solar systems, which used to be a "back-to-the-land" phenomenon in the developed world, are now seen by some to be the market of the future in emerging economies around the globe. Some estimates suggest there are 150 GW of diesel generators out there providing power at over $0.40/kWh that could be replaced with renewables (Bloomberg New Energy Finance, 2012).

6.5.1 Stand-alone System Components

Figure 6.21 identifies a number of important components that make up a basic stand-alone PV system. The system consists of the PV array itself, batteries for storage, a charge controller, an inverter, and a system monitor, along with a number of disconnect switches. A backup generator may or may not be included in the system.

FIGURE 6.21 A "one-line"diagram of the important components in a stand-alone PV system.



The PV array delivers power through its accompanying combiner box to a controller. The controller has three important functions. Its MPPT keeps the PVs operating at their most efficient operating point. Its charge control function protects the batteries by shutting off charging current when they are fully charged and by disconnecting the batteries from the DC loads when it senses a low-voltage condition.

The system meter provides real-time information about the performance of the system. It is not essential, but it is very important. Depending on their capabilities, they can monitor battery voltage, current being delivered to and from the batteries, and the state of charge (SOC) of the batteries. It may also indicate the accumulated energy flows from the battery system. The diagram shows a current shunt, which is a very precise in-line resistor with a very small resistance. The system meter measures current flows by monitoring the voltage drop across the shunt.

Sometimes, especially with smaller systems, the only loads might be just DC-powered devices, in which case the inverter may not be necessary. In some circumstances, both AC and DC loads may be provided for from the same system. Some systems do so to avoid inverter losses whenever possible by powering as many loads as possible on DC. Quite a range of DC devices are manufactured for boating and recreational vehicles, so they can be quite readily available. Another reason for providing both AC and DC is to allow some high power, low usage devices such as water pumps or shop motors to run on DC, thereby avoiding the need to install an overly large and expensive inverter that will not perform well when only modest loads are supplied. Most inverters have efficiencies over 90% as long as they are not operated well below their rated power, but when an inverter rated at several kW is delivering only 100 W, its efficiency may be more like 60–70%. When no load is present, a good inverter will power down to just a few watts of standby power while it waits for something to be turned on that needs AC. When it senses a load, the inverter powers up and while running uses on the order of 5–20 W of its own. That means standby losses associated with so many of our electronic devices may keep the inverter running continuously, even though no real energy service is being delivered. That much loss for so little gain suggests paying attention to manually shutting down turned-off electronic equipment.

There are other features that can come into play with inverters. An important attribute is a low-voltage disconnect to protect the batteries. Also, as shown in Figure 6.21, some inverters are bidirectional, which means they can also act as battery chargers when a backup AC engine–generator is part of the system. As a charger, it converts AC from the generator into DC to charge the batteries; as an inverter it converts DC from the batteries into AC needed by the load. The charger/inverter unit may include an automatic transfer switch that allows the generator to supply AC loads directly whenever it is running.

Minor, but important, players in these systems are the fuses, circuit breakers, cables, wires and wire terminals, mounting hardware, battery boxes, earth-grounding and lighting-protection systems, and so forth. Unfortunately, far too many systems installed in rural areas have failed prematurely due to lack of attention being paid to these details. For good summaries of lessons learned designing and installing real systems, see, for example, Undercuffler (2010) and Youngren (2011).

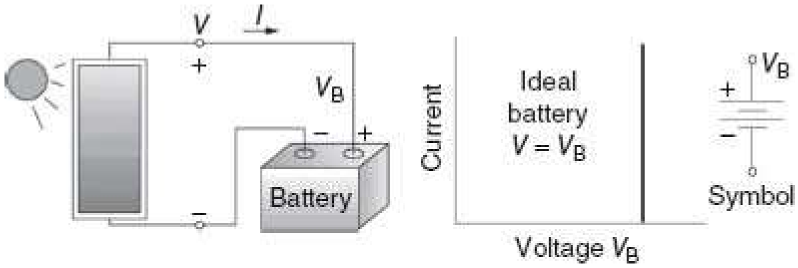
Off-grid systems must be designed with great care to assure satisfactory performance. Users must be willing to check and maintain batteries, they must be willing to adjust their energy demands as weather and battery charge vary, they may have to fuel and fix a balky generator, and they must take responsibility for the safe operation of the system. The reward is electricity that is truly valued.

6.5.2 Self-regulating Modules

Figure 6.21 describes an ideal system capable of delivering enough power to handle significant loads. In some circumstances, however, even simpler systems can make great sense. Staying in DC eliminates the inverter. Directly coupling PVs to the batteries without an MPPT shaves the cost. A simple charge controller to help protect the batteries along with appropriate fuses, breakers, and wiring can complete the system.

Let us analyze the possibility of eliminating the charge controller altogether. Consider the minimal system shown in Figure 6.22 in which a module is directly connected to a battery. Begin by assuming an ideal battery in which the voltage remains constant no matter what its state of charge or the rate at which it is being charged or discharged. That means it will have an *I*–*V* curve that is simply a straight up-and-down line as shown in the figure.

FIGURE 6.22 An ideal battery has a vertical *I*–*V* characteristic curve.



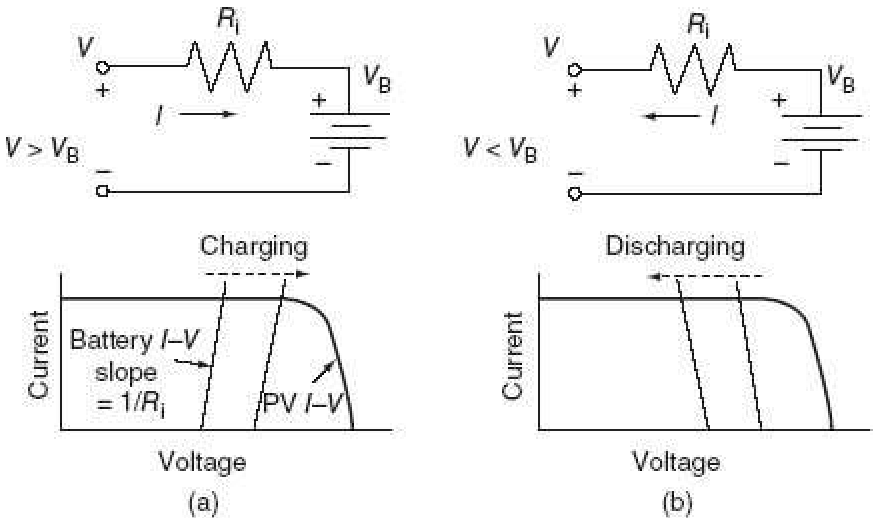
The simple equivalent circuit representation of Figure 6.22 is complicated by a number of factors, including the fact that the open-circuit voltage V_B depends not only on the state of charge, but also on battery temperature and how long it has been resting without any current flowing. For a conventional 12-V, lead–acid battery at 78°F, which has been allowed to rest for a day, V_B ranges from 12.7 V for a fully charged battery to about 11.3 V for one that has only 10% of its charge remaining.

A real battery also has some internal resistance, which is often modeled with an equivalent circuit consisting of an ideal battery of voltage V_B in series with the internal resistance R_i as shown in Figure 6.23. During the charge cycle, with positive current flow into the battery, we can write

(6.20) $V = V_B + R_i I$

which plots as a slightly tilted, straight line with slope equal to I/R_i. During charging, the applied voltage needs to be greater than V_B. As the process continues, V_B itself increases so the *I*–*V* line slides to the right as shown in Figure 6.23a. During discharge, the output voltage of the battery is less than V_B, the slope of the *I*–*V* line flips, and the *I*–*V* curve moves back to the left as shown in Figure 6.23b. Even this rendition can be further refined to account for variations in the internal resistance as a function of temperature and state of charge, as well the age and condition of the battery.

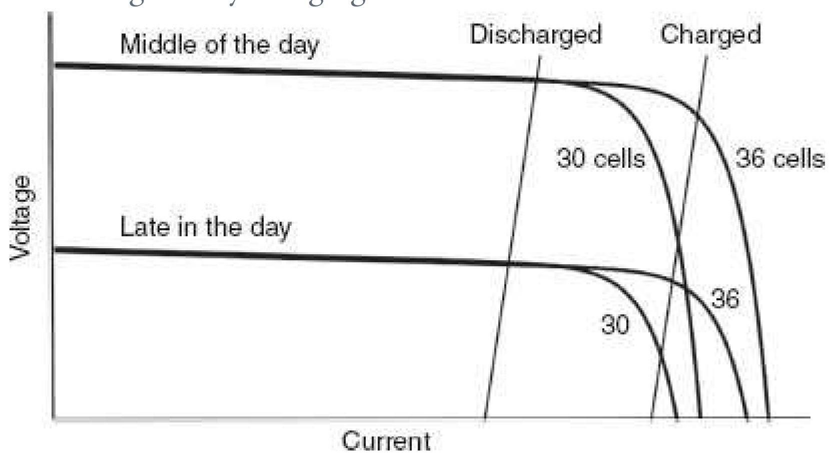
FIGURE 6.23 A real battery can be modeled as an ideal battery in series with its internal resistance, with current flowing in opposite directions during charging (a) and discharging (b). During charging/discharging, the slightly tilted *I*–*V* curve slides right or left.



The operating point of the battery–PV combination is the spot at which both have the same current and voltage; that is, it is the intersection of the two *I–V* curves. Since both *I–V* curves vary with time, predicting the performance of this simple system can be a challenge. For example, since the *I–V* curve for a battery moves toward the right as the battery gains charge during the day, there is a chance that the PV operating point will begin to slide off the edge of the knee—especially late in the day when warmer temperatures and lower insolation cause the knee itself to move toward the left.

Sliding off the knee in the afternoon may not be a bad thing, however, since current has to be slowed or stopped anyway when a battery reaches full charge. If the PV-battery system has a charge controller, it will automatically prevent overcharging of the batteries. For very small battery charging systems, however, the charge controller can sometimes be omitted if modules with fewer cells in series are used. Such self-regulating modules sometimes have 33, or even 30, cells instead of the usual 36 that normal 12-V battery charging PVs typically have. With 30 cells, V_{MPP} is around 14 V and V_{OC} is about 18 V (compared to the 17 V and 21 V values for a 36-cell module). The idea is to purposely cause the current to drop off as the battery approaches full charge as suggested in Figure 6.24. There is some risk with this approach and since it does not protect against over discharging, a simple and inexpensive charge controller would be preferred.

FIGURE 6.24 A self-regulating PV module with fewer cells offers a risky approach to automatically controlling battery charging.



6.5.3 Estimating the Load

The design process for stand-alone systems begins with an estimate of the loads that are to be provided for. As with all design processes, a number of iterations may be required. On the first pass, the user may try to provide the capability to power anything and everything that normal, grid-connected living allows. Various iterations will follow in which trade-offs are made between more expensive, but more efficient, appliances and devices in exchange for fewer PVs and batteries. Lifestyle adjustments need to be considered in which some loads are treated as essentials that must be provided for and others are luxuries to be used only when conditions allow. A key decision involves whether to use all DC loads to avoid the inefficiencies associated with inverters, or whether the convenience of an all AC system is worth the extra cost, or perhaps a combination of the two is best. Another important decision is whether to include a generator backup system, and if so, what fraction of the load it will have to supply.

Power needed by a load and energy required over time by that load are both important for system sizing. In the simplest case, energy (watt-hours or kilowatt-hours) is just the product of some nominal power rating of the device times the hours that it is in use. The situation is often more complicated, however. For example, an amplifier needs more power when the volume is increased, and many appliances, such as refrigerators and washing machines, use different amounts of power during different portions of their operating cycles. An especially important consideration for household electronic devices—TVs, VCRs, computers, portable phones, and so on—is the power consumed when the device is in its standby or charging modes. Many devices, such as TVs, use power even while they are turned off since some circuits remain energized awaiting the turn-on signal from the remote. Other devices, especially cable and dish video recording boxes tend to consume almost the same amount of power 24 h a day as they update their TV guides and await the shows that you have programmed to record. Consumer electronics now account for about 10% of all U.S. residential electricity and researchers at Lawrence Berkeley National Labs conclude that almost two-thirds of that energy occurs when these devices are not actually being used (Rosen and Meier, 2000). Major appliances and shop tools have another complication caused by the surge of power required to start their electric motors. While that large initial spike does not add much to the energy used by a motor, it has important implications for sizing inverters, wires, fuses, and other ancillary electrical components in the system.

Table 6.11 lists examples of power used by a number of household electrical loads. Some of these are simply watts of power, which can be multiplied by hours of use to get watt-hours of energy. Many of the devices listed in the consumer electronics category show power while they are being used (active) and power consumed the rest of the time (standby), both of which must be considered when determining energy consumption. Refrigerators are also unusual since they are always turned on, but their power demand varies throughout the day. The data given on refrigerator labels are average watt-hours per day based on measurements made with the refrigerator located in a 90°F room. That means they are likely to overstate the actual demand in someone's home—perhaps by as much as 20%. Tables like this one are useful for average values, but the best source of power and energy data are actual measurements that can easily be performed with readily available meters. Another source is the device nameplate itself, but those tend to overstate power since they are meant to describe maximum demand rather than the likely average. Some nameplates provide only amperage and voltage, and while it is tempting to multiply the two to get power, that can also be an overestimate since it ignores the phase angle, or power factor, between current and voltage.

TABLE 6.11 Power Requirements of Typical Household Loads

Kitchen Appliances	
Refrigerator/freezer: Energy Star 14 cu ft.	300 W, 950 Wh/d
Refrigerator/freezer: Energy Star 19 cu ft.	300 W, 1080 Wh/d
Refrigerator/freezer: Energy Star 22 cu ft.	300 W, 1150 Wh/d
Chest freezer: Energy Star 22 cu ft.	300 W, 1300 Wh/d
Dishwasher (hot dry)	1400 W, 1.5 kWh/load
Electric range burner (small/large)	1200 / 2000 W
Toaster oven	750 W
Microwave oven	1200 W
General Household	
Clothes dryer (gas/electric, 1400 W)	250 W; 0.3 / 3 kWh/load
Washer (w/o H_2O heating/with electric heating)	250 W; 0.3 / 2.5 kWh/load
Furnace fan: $^1/_2$ hp	875 W
Celling fan	100 W
Air conditioner: window, 10, 000 Btu	1200 W
Heater (portable)	1200–1875 W
Compact fluorescent lamp (100 W equivalent)	25 W
Clothes iron	1100 W
Clocks, cordless phones, answering machines	3 W
Hair dryer	1500 W
Consumer electronics (Active/Standby)	
TV: 30–36 in Tube	120 / 3.5 W
TV: 40–49 in Plasma	400 / 2 W
TV: 40–49 in LCD	200 / 2 W
Satellite or cable with DVR (Tivo)	44 / 43 W
Digital cable box (no DVR)	24 / 18 W
DVD, VCR	15 / 5 W
Game console (X-Box)	150 / 1 W
Stereo	50 / 3 W
Modem DSL	5 / 1 W
Printer inkjet	9 / 5 W
Printer laser	130 / 2 W
Tuner AM/FM	10 / 1 W

Computer: Desktop (on/sleep/off)	74 / 21 / 3 W
Computer: Notebook (in use/sleep)	30 / 16 W
Computer monitor LCD	40 / 2 W
Outside	
Power tools, cordless	30 W
Circular saw, 7 1/4 in	900 W
Table saw, 10 in	1800 W
Centrifugal water pump: 50 ft at 10 gal/min	450 W
Submersible water pump: 300 ft at 1.5 gal/min	180 W
Source: Rosen and Meier (2000) with updates from LBL and others.	

Example 6.9 A Modest Household Demand. Estimate the monthly energy demand for a cabin with all AC appliances, consisting of a 19 cu ft refrigerator, six 25-W compact fluorescent lamps (CFLs) used 6 h/d, a 44-in LCD TV turned on 3 h/d and connected to a satellite with digital video recording (DVR), 10 small electric devices using 3 W continuously, a microwave used 12 min/d and a small range burner 1 h/d, a clothes washer that does four loads a week with solar heated water, a laptop computer used 2 h/d, and a 300-ft deep well that supplies 120 gal of water per day.

Solution. Using data from Table 6.11, we can put together the following table of power and energy demands. The total is about 6.3 kWh/d, which is about 2300 kWh/yr.

Appliance	Power (W)	Hours/day	Wh/d	Percentage
Refrigerator, 19 cu ft	300		1080	17%
Range burner (small)	1200	1	1200	19%
Microwave at 12 min/d	1200	0.2	240	4%
Lights (6 at 25 W, 6 h/d)	150	6	900	14%
Clothes washer (4 load/wk at 0.3 kWh)	250		171	3%
LCD TV 3 h/d (on)	200	3	600	10%
LCD TV 21 h/d (standby)	2	21	42	1%
Satellite with DVR	44	3	132	2%
Satellite (standby)	43	21	903	14%
Laptop computer (2 h/d at 30 W)	30	2	60	1%
Assorted electronics (10 at 3 W)	30	24	720	11%
Well pump (120 gal/d at 1.5 gal/min)	180	1.33	240	4%
Total	3566		6288	

We can note several interesting things from this table. For starters, kitchen use accounts for 40% of the total, which is a relatively high fraction due in part to an assumption that it is an all-electric stove. On the other hand, this is a very good refrigerator that uses only about half the energy of an old one. When using PV power, it will almost always make sense to purchase the most efficient appliances available.

Watching TV accounts for 27% of the energy, with 55% of that being standby loads mostly associated with that DVR.

6.5.4 Initial Array Sizing Assuming an MPP Tracker

As should be the case in most design processes, a good start is based on many assumptions that will have to be refined as the design proceeds. Our starting point for array sizing will be based on the assumption that the system includes an MPPT and that the batteries will be sized to carry the load during inclement weather conditions. The first assumption allows us to use the peak-hours approach to system sizing and the second allows us to use average solar insolation values instead of hour-by-hour TMY calculations.

Without the grid, the orientation of the collectors becomes much more important since we cannot carry much, if any, excess power from 1 month to the next. That suggests starting with a fairly steep collector tilt angle to try to bump up the winter insolation and sizing the system based on the worst month of the year.

Example 6.10 Initial Array Sizing for a House in Boulder, CO. Using Appendix G insolation data, size an off-grid array to meet the 6.29 kWh/d load found in Example 6.9. For now, assume an 80% round-trip battery efficiency and make any other appropriate assumptions.

Solution. To get relatively uniform output over the year we will chose south-facing, L+15 tilt angle collectors. From Appendix G, the month with the least insolation is December with 4.5 kWh/m^2/d, or 4.5 h/d of full sun. Let us assume the usual 0.75 derate factor for inverter, dirt, wiring, and so on We will assume all of the PV kW go into and back out of the battery on their way to the load, so we will apply the full 80% round-trip efficiency:

$$6.29\ \text{kWh/d} = P_{DC}\ (\text{kW}) \cdot 4.5\ \text{h/d} \cdot 0.75 \cdot 0.80$$

$$P_{DC} = \frac{6.29}{4.5 \times 0.75 \times 0.80} = 2.33\ \text{kW}$$

That takes care of December, but it will mean excess energy for the other months of the year. Repeating the calculation leads to the following monthly results:

	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec	Annual
Insolation (h/d)	4.8	5.3	5.6	5.6	5.2	5.2	5.3	5.5	5.8	5.7	4.8	4.5	5.3
PV (kWh/d)	6.71	7.41	7.83	7.83	7.27	7.27	7.41	7.69	8.11	7.97	6.71	6.29	2691
Load (kWh/d)	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	2295
Excess	0.42	1.12	1.54	1.54	0.98	0.98	1.12	1.40	1.82	1.68	0.42	0.00	396

These are average monthly values, which include inclement weather periods during which we hope there is enough storage to carry us through, or else behavioral modifications that will reduce the load appropriately. And there is always the option of a generator backup.

Suppose we change the design criteria to specify that the PVs will cover the average annual load, in which case the rated power of the array would be

$$P_{DC} = \frac{6.29}{5.3 \times 0.75 \times 0.80} = 1.98\ \text{kW}$$

While this could provide all of the loads on the average, it is unlikely to do so without an excessively large storage system to carry excess from 1 month into a later month with deficits. Consider the following table of performance for the 1.98-kW system, in which no month-to-month carryover from good months to bad months is counted.

	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec	Annual
Insolation (h/d)	4.8	5.3	5.6	5.6	5.2	5.2	5.3	5.5	5.8	5.7	4.8	4.5	5.3
PV (kWh/d)	5.72	6.29	6.29	6.29	6.20	6.20	6.29	6.29	6.29	6.29	5.72	5.36	2227
Load (kWh/d)	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	6.29	2295

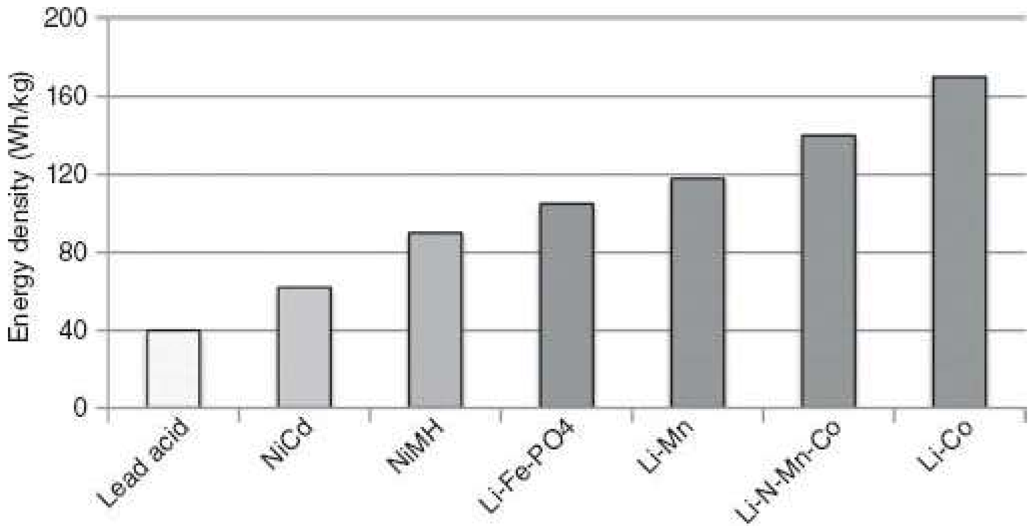
The 1.98-kW system is only 2295 − 2227 = 68 kWh/yr short of meeting the load while saving 2.33 − 1.98 = 0.35 kW of PV costs, which at say $4/W would save about $1400. The marginal cost of those last 68 kWh/yr would be hard to justify, especially if a backup generator becomes part of the system, which suggests the smaller system might provide better value.

As we shall see, when PVs are directly connected to batteries without an MPP tracker, the simple "peak-hour" approach for estimating the interaction between generation and loads no longer applies. To explore that, we need to learn more about batteries.

6.5.5 Batteries

Stand-alone systems obviously need some method to store energy gathered during good times to be able to use it during the bad. Depending on scale, a number of technologies are available now, or will be in the near future. Possible storage systems include flow batteries, compressed air, pumped storage, flywheels, and electrolysis of water to make hydrogen for fuel cells. For simple off-grid systems, however, it is still the lowly battery that makes the most sense today. And, among the many possible battery technologies, it is the familiar lead–acid battery that continues to be the workhorse of PV systems. The main competitor to conventional lead–acid batteries are various rapidly emerging lithium-ion technologies that power most of today's electric vehicles. The far greater energy density (Wh/kg) of lithium batteries is much more important for vehicles than for stationary applications, but as their costs decrease, they will likely become the battery of choice for off-grid PV systems as well in the near future. Figure 6.25 shows how much more energy can be packed into small packages with various lithium-ion battery technologies.

FIGURE 6.25 Lead–acid batteries are bigger and heavier than their emerging competition, but they remain the least costly option (from Battery University website, 2012).



In addition to energy storage, batteries provide several other important energy services for PV systems, including the ability to provide surges of current that are much higher than the instantaneous current available from the array, as well as the inherent and automatic property of controlling the output voltage of the array so that loads receive voltages that are within their own range of acceptability.

6.5.6 Basics of Lead–Acid Batteries

Lead–acid batteries date back to the 1860s when inventor Raymond Gaston Planté fabricated the first practical cells made with corroded lead-foil electrodes and a dilute solution of sulfuric acid and water. Automobile SLI batteries (starting, lighting, ignition) have been highly refined to perform their most important task, which is to start your engine. To do so, they have to provide short bursts of very high current (400–600 A). Once the engine has started, its alternator quickly recharges the battery, which means under normal circumstances the battery is almost always at or near full charge. SLI batteries are not designed to withstand deep discharges, and in fact will fail after only a few complete discharge cycles. That makes them inappropriate for most PV systems, in which slow, but deep, discharges are the norm. If they must be used, as is sometimes the case in developing countries where they may be the only batteries available, daily discharges of less than about 25% can yield several hundred cycles, or a year or two of operation at best.

In comparison with SLI batteries, deep discharge batteries have thicker plates, which are housed in bigger cases that provide greater space both above and beneath the plates. Greater space below allows more debris to accumulate without shorting out the plates, and greater space above lets there be more electrolyte in the cell to help keep water losses from exposing the plates. Thicker plates and larger cases mean these batteries are big and heavy. A single 12-V deep discharge battery can weigh several hundred pounds. They are designed to be discharged repeatedly by 80% of their capacity without harm, although such deep discharges result in a lower lifetime number of cycles. A deep-cycle, lead–acid battery can be cycled several thousand times with daily discharges of 25% or less of its rated capacity, which would give it a lifetime on the order of 10 years. That lifetime could be cut in half with 50% daily discharges, which suggests the battery bank in a PV system should be designed to store at least 4 or 5 days worth of energy demand to minimize deep cycling and prolong battery life.

To understand some of the subtleties in sizing battery systems, we need a basic understanding of their chemistry. Very simply, an individual cell in a lead–acid battery consists of a positive electrode made of lead dioxide (PbO_2) and a negative electrode made of a highly porous, metallic lead (Pb) structure, both of which are completely immersed in a sulfuric acid electrolyte. Thin lead plates are structurally very weak and would not hold up well to physical abuse unless alloyed with a strengthening material. Automobile SLI batteries use calcium for strengthening, but calcium does not tolerate discharges of more than about 25% very well. Deep discharge batteries use antimony instead, and so are often referred to as lead–antimony batteries.

One way to categorize lead–acid batteries is based on whether they are sealed or not. Conventional vehicle batteries are flooded, which means the plates are submerged in a weak solution of sulfuric acid and water. Toward the end of a charging cycle, voltage may rise enough to cause electrolysis, which releases potentially dangerous hydrogen and oxygen gases while it removes water from the battery. Flooded batteries are vented to allow these gases to escape and they require access to the cells to maintain proper water level over the plates.

Sealed batteries, on the other hand, have special valves to minimize gas releases by internally recycling those gases within the battery itself, hence the name valve-regulated lead–acid (VRLA) batteries. Chargers for sealed batteries need to be designed to avoid overvoltages, which are the source of battery outgassing. The electrolyte in sealed batteries may take the form of a gel or as an absorbent glass mat (AGM). They are a bit more expensive, but eliminating the need for water maintenance, along with their ability to be oriented in any direction without spillage, makes them a common choice for PV systems.

The chemical reactions taking place while a lead–acid battery discharges are as follows:

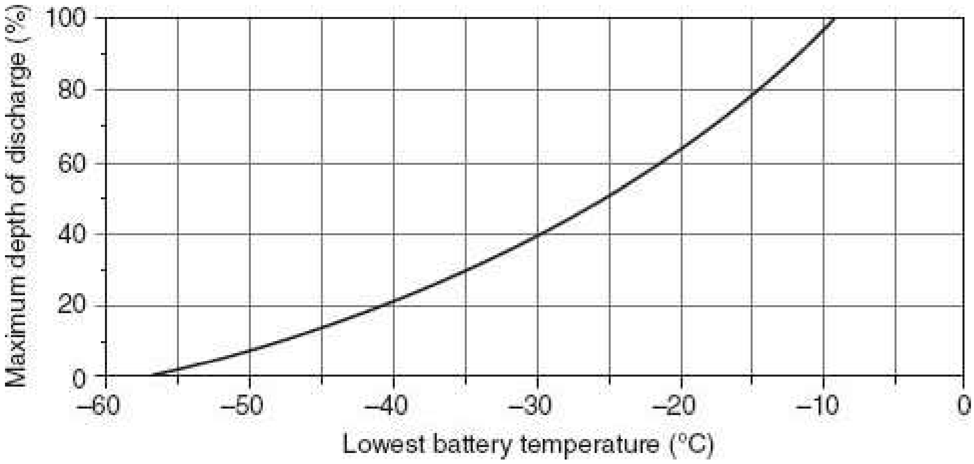
(6.21) $$\text{Positive plate: } PbO_2 + 4H^+ + SO_4^{2-} + 2e^- \rightarrow PbSO_4 + 2H_2O$$

(6.22) $$\text{Negative plate: } Pb + SO_4^{2-} \rightarrow P_bSO_4 + 2e^-$$

It is, by the way, simpler to refer to the terminals by their charge (positive or negative) rather than as the anode and cathode. Strictly speaking, the anode is the electrode at which oxidation occurs, which means during discharge the anode is the negative terminal, but during charging it is the positive terminal.

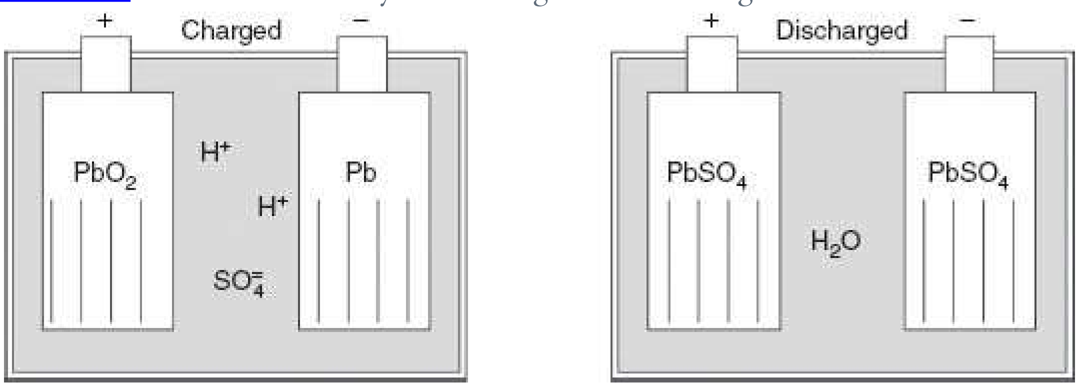
As can be seen from Equation 6.22, during discharge electrons are released at the negative electrode, which then flow through the load to the positive plate where they enter into the reaction given by Equation 6.21. The key feature of both reactions is that sulfate ions (SO_4^{2-}) that start out in the electrolyte when the battery is fully charged, end up being deposited onto each of the two electrodes as lead sulfate ($PbSO_4$) during discharge. That lead sulfate, which is an electrical insulator, blankets the electrodes leaving less and less active area for the reactions to take place. As the battery approaches its fully discharged state, the cell voltage drops sharply while its internal resistance rises abruptly. Meanwhile, during discharge, the specific gravity of the electrolyte drops as sulfate ions leave solution, providing an accurate indicator of the battery's state of charge. The battery is more vulnerable to freezing in its discharged state since the antifreeze action of the sulfuric acid is diminished when there is less of it present. A fully discharged lead–acid battery will freeze at around −8°C (17°F), while a fully charged one will not freeze until the electrolyte drops below −57°C (−71°F). In very cold conditions, concern for freezing may limit the maximum allowable depth of discharge, as shown in Figure 6.26.

FIGURE 6.26 Concern for battery freezing may limit the allowable depth of discharge of a lead–acid battery.



The opposite reactions occur during charging. Battery voltage and specific gravity rise, while freeze temperature and internal resistance drops. Sulfate is removed from the plates and re-enters the electrolyte as sulfate ions (Fig. 6.27). Unfortunately, not all of the lead sulfate returns to solution and each battery charge/discharge cycle leaves a little more sulfate permanently attached to the plates. This sulfation is a primary cause of a battery's finite lifetime. The amount of lead sulfate that permanently bonds to the electrodes depends on the length of time that it is allowed to exist, which means for good battery longevity, it is important to keep them as fully charged as possible and to completely charge them on a regular basis. That suggests a generator backup system to top up batteries is an important consideration.

FIGURE 6.27 A lead–acid battery in its charged and discharged states.



As batteries cycle between their charged and partially discharged states, the voltage as measured at the terminals and the specific gravity of the electrolyte changes. While either may be used as an indication of the SOC of the battery, both are tricky to measure correctly. To make an accurate voltage reading, the battery must be at rest, which means at least several hours must elapse after any charging or discharging. Specific gravity is also difficult to measure since stratification of the electrolyte means a sample taken from the liquid above the plates may not be an accurate average value. Since sealed batteries are now more common for PV systems, SOC is most easily estimated by a simple measurement of the at-rest (preferably for at least 6 h), open-circuit battery voltage. The following is an estimate for the state of charge as a function of the at-rest, open-circuit voltage (V_{OC}) for a 12-V lead–acid battery (based on Trojan Battery Company data):

(6.23) $$SOC(\%) = 73.1 \cdot V_{OC} - 833.3$$

For example, if the measured battery voltage is 12 V, the SOC would be

$$SOC(\%) = 73.1 \times 12 - 833.3 = 44\%$$

while at full charge the voltage would be a bit over 12.7 V.

6.5.7 Battery Storage Capacity

Energy storage in a battery is typically given in units of ampere-hours (Ah) at some nominal voltage and at some specified discharge rate. A lead–acid battery, for example, has a nominal voltage of 2 V per cell (e.g., 6 cells for a 12-V battery) and manufacturers typically specify the ampere-hour capacity at a discharge rate that would drain the battery down over a specified period of time at a temperature of 25°C. For example, a fully charged 12-V battery that is specified to have a 20-h, 200-Ah capacity could deliver 10 A for 20 h, at which point the battery would be considered to be fully discharged. This ampere-hour specification is referred to as a C/20 or 0.05C rate, where the C refers to ampere-hours of capacity and the 20 is hours it would take to deplete. Note how tricky it would be to specify how much energy the battery delivered during its discharge. Energy is volts × amperes × hours, but since voltage varies throughout the discharge period, we cannot just say 12 V × 10 A × 20 h = 2400 Wh. To avoid that ambiguity, almost everything having to do with battery storage capacity is specified in ampere-hours rather than watt-hours.

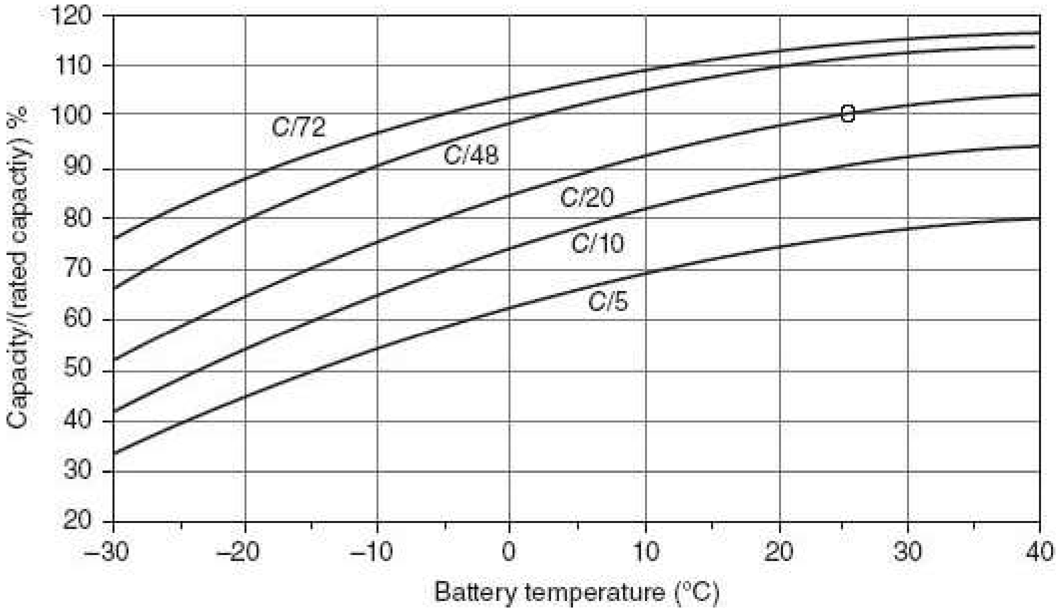
The ampere-hour capacity of a battery is very much tied to the discharge rate. More rapid drawdown of a battery results in lower ampere-hour capacity, while longer discharge times result in higher ampere-hour capacity. Deep-cycle batteries intended for PV systems are often specified in terms of their 20- or 24-h discharge rate as well as the much longer C/100 rate that is more representative of how they are actually used. Table 6.12 provides some examples of such batteries, including their C/20 rates as well as their voltage and weight.

TABLE 6.12 Example Deep-Cycle Lead–Acid Battery Characteristics

Battery	Electrolyte	Voltage	Nominal Ah	Rate (h)	Weight (lbs)
Rolls Surette 4CS-17P	Flooded	4	546	20	128
Trojan T10S-RE	Flooded	6	225	20	67
Concorde PVX 3050T	AGM	6	305	24	91
Fullriver DC260-12	AGM	12	260	20	172
Trojan 5SHP-GEL	Gel	12	125	20	85

The ampere-hour capacity of a battery is not only rate dependent, but it also depends on temperature. Figure 6.28 captures both of these phenomena by comparing capacity under varying temperature and discharge rates to a reference condition of C/20 and 25°C. These curves are approximate for typical deep-cycle lead–acid batteries, but specific data available from the battery manufacturer should be used whenever possible. As shown in the figure, battery capacity decreases dramatically in colder conditions. At −30°C (−22°F), for example, a battery that is discharged at the C/20 rate will have only half of its rated capacity. The combination of cold temperature effects on battery performance—decreased capacity, decreased output voltage, and increased vulnerability to freezing when discharged—means that lead–acid batteries need to be well protected in cold climates. Nickel–cadmium batteries do not suffer from these cold weather effects, which is the main reason they are sometimes used instead of lead–acids in cold climates. By the way, the apparent improvement in battery capacity at high temperatures does not mean heat is good for a battery. In fact, a rule-of-thumb estimate is that battery life is shortened by 50% for every 10°C above the optimum 25°C operating temperature.

FIGURE 6.28 Lead–acid battery capacity depends on discharge rate and temperature. The capacity percentage ratio is based on a rated capacity at C/20 and 25°C.



Example 6.11 Battery Storage Calculation in a Cold Climate. Suppose batteries located at a remote telecommunication site may drop to –20°C. If they must provide 2 days of storage for a load that needs 500 Ah/d at 12 V, how many ampere-hours of storage should be specified for the battery bank?

Solution. From Figure 6.26, to avoid freezing the maximum depth of discharge at –20°C is about 60%. For 2 days of storage, with a discharge of no more than 60%, the batteries need to store

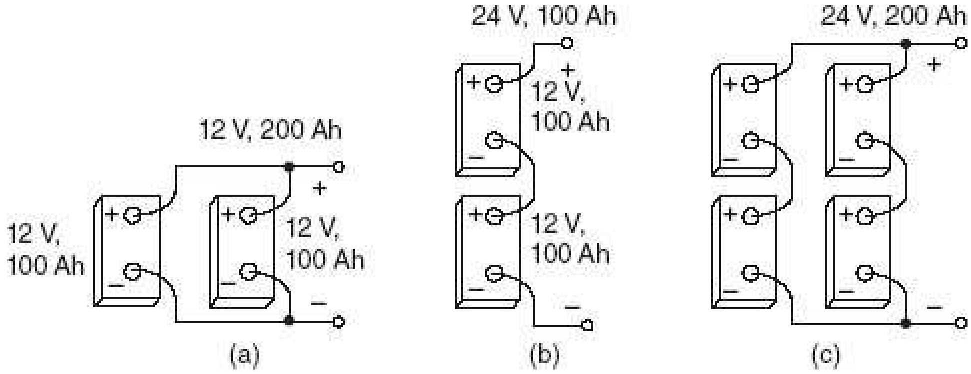
$$\text{Battery storage} = \frac{500\ \text{Ah/d} \times 2\ \text{days}}{0.60} = 1667\ \text{Ah}$$

Since the rated capacity of batteries is likely to be specified at an assumed temperature of 25°C at a C/20 rate, we need to adjust the battery capacity to account for our different temperature and discharge period. From Figure 6.28, the actual capacity of batteries at –20°C discharged over a 48-h period is about 80% of their rated capacity. That means we need to specify batteries with rated capacity

$$\text{Battery storage}\,(25^\circ\,\text{C, 48-h rate}) = \frac{1667\ \text{Ah}}{0.8} = 2082\ \text{Ah}$$

Battery systems typically consist of a number of batteries wired in series and parallel combinations to achieve the needed ampere-hour capacity and voltage rating. For batteries wired in series, the voltages add, but since the same current flows through each battery, the ampere-hour rating of the string is the same as it is for each battery. For batteries wired in parallel, the voltage across each battery is the same, but since currents add, the ampere-hour capacity is additive. Figure 6.29 illustrates these notions.

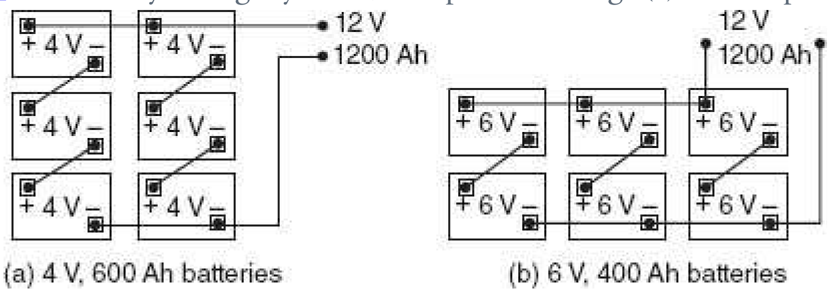
FIGURE 6.29 For batteries wired in parallel, ampere-hours add (a). For batteries in series, voltages add (b). For series/parallel combinations, both add (c).



Since there is no difference in energy stored in the 2-battery, series and parallel example shown in Figure 6.29, the question arises as to which is better. The key difference between the two is the amount of current that flows to deliver a given amount of power. Batteries in series have higher voltage and lower current, which means more manageable wire sizes without excessive voltage and power losses, along with smaller fuses and switches, and slightly easier connections between batteries. So Figure 6.29b is preferred over Figure 6.29a.

Once the system voltage for the battery bank has been determined, with higher being better, then there is a tradeoff between series and parallel combinations of batteries to achieve the desired total ampere-hours of storage. When batteries are wired in parallel, the weakest battery will drag down the voltage of the entire bank, so the most efficient battery configuration is one that has all of the batteries wired in a single series string. However, if a battery fails in a single-string system, the entire bank can go down. For redundancy, then, good practice suggests two parallel strings so that the user is never completely without power. Thus, in the examples shown in Figure 6.30, the configuration in (a) is preferred.

FIGURE 6.30 For battery storage systems, two parallel strings (a) are the preferred option.



6.5.8 Coulomb Efficiency Instead of Energy Efficiency

As mentioned earlier, almost everything having to do with batteries is described in terms of currents rather than voltage or energy. Battery capacity C is given in ampere-hours rather than watt-hours, charging and discharging is expressed in C/T rates, which are amperes. And, as we shall see, even battery efficiency is more easily expressed in terms of current efficiency rather than energy efficiency. The reason, of course, is that battery voltage is so ambiguous without specifying whether it is a "rest" voltage measured some time after charging or discharging, a voltage during charging, or a voltage during discharge. And even those charging and discharging voltages depend on the rates at which current is entering or leaving the battery as well as the state of charge of the battery, its temperature, age, and general condition.

Imagine charging a battery with a constant current I_C over a period of time ΔT_C during which time applied voltage is V_C. The energy input to the battery is thus

(6.24) $$E_{in} = V_C I_C \Delta T_C$$

Suppose the battery is then discharged at current I_D, voltage V_D over a period of time ΔT_D, delivering energy

(6.25) $$E_{out} = V_D I_D \Delta T_D$$

The energy efficiency of the battery would be

(6.26) $$\text{Energy efficiency} = \frac{V_D I_D \Delta T_D}{V_C I_C \Delta T_C}$$

If we recognize that current (A) × time (h) is Coulombs of charge expressed as Ah

(6.27) $$\text{Energy efficiency} = \left(\frac{V_D}{V_C}\right)\left(\frac{I_D \Delta T_D}{I_C \Delta T_C}\right) = \left(\frac{V_D}{V_C}\right)\left(\frac{\text{Coulombs out, Ah}_{out}}{\text{Coulombs in, Ah}_{in}}\right)$$

The ratio of discharge voltage to charge voltage is called the *voltage efficiency* of the battery, and the ratio of Ah_{out} to Ah_{in} is called the *Coulomb efficiency*.

(6.28) $$\text{Energy efficiency} = (\text{Voltage efficiency}) \times (\text{Coulomb efficiency})$$

A typical 12-V lead–acid battery might be charged at a voltage of around 14 V and its discharge voltage might be around 12 V. Its voltage efficiency would therefore be

(6.29) $$\text{Voltage efficiency} = \frac{12\ \text{V}}{14\ \text{V}} = 0.86 = 86\%$$

The Coulomb efficiency is the ratio of coulombs of charge out of the battery to coulombs that went in. If they all do not come back out, where did they go? When a battery approaches full charge, its cell voltage gets high enough to electrolyze water, creating hydrogen and oxygen gases that may be released. Among the negative effects of this *gassing* is loss of some of those charging electrons along with the escaping gases. As long as the battery SOC is low, little gassing occurs and the Coulomb efficiency is nearly 100%, but it can drop below 90% during the final stages of charging. Over a full charge cycle, it is typically 90–95%. As we shall see later, when it comes to sizing batteries, the Coulomb efficiency will be the measure that is most appropriate.

Assuming a 90% Coulomb efficiency, the overall energy efficiency of a lead–acid battery with 86% voltage efficiency would be about

$$\text{Energy efficiency} = 0.86 \times 0.90 = 0.77 = 77\% \quad (6.30)$$

which is close to a commonly quoted estimate of 75% for lead–acid battery efficiency.

6.5.9 Battery Sizing

Two design decisions have to be made when sizing batteries for a stand-alone system. Instead of working with energy storage in kWh, it is the voltage of the battery bank and the ampere-hour rating of individual batteries in the array that matter.

The voltage of the battery bank has to be matched to the input voltage requirements of the inverter (if there is one) or the loads themselves if it is an all-DC system. To control I^2R wire losses, higher voltages are preferred. Lower currents means smaller gauge wire can be used, which is easier to work and, along with the associated breakers, fuses, and other wiring details, is cheaper. The system voltage for modest loads is usually 12, 24, or 48 V. One guideline that can be used to pick the system voltage is based on keeping the maximum steady-state current drawn below around 100 A, so that readily available electrical hardware and wire sizes can be used. Using this guideline results in the minimum system voltage suggestions given in Table 6.13. Maximum steady-state power that the batteries and inverter have to supply can be estimated by adding the individual power demands of all the loads that might be expected to be operating at the same time. Table 6.11 provided representative values of power for a number of common household devices, which can be used as a starting point in an analysis.

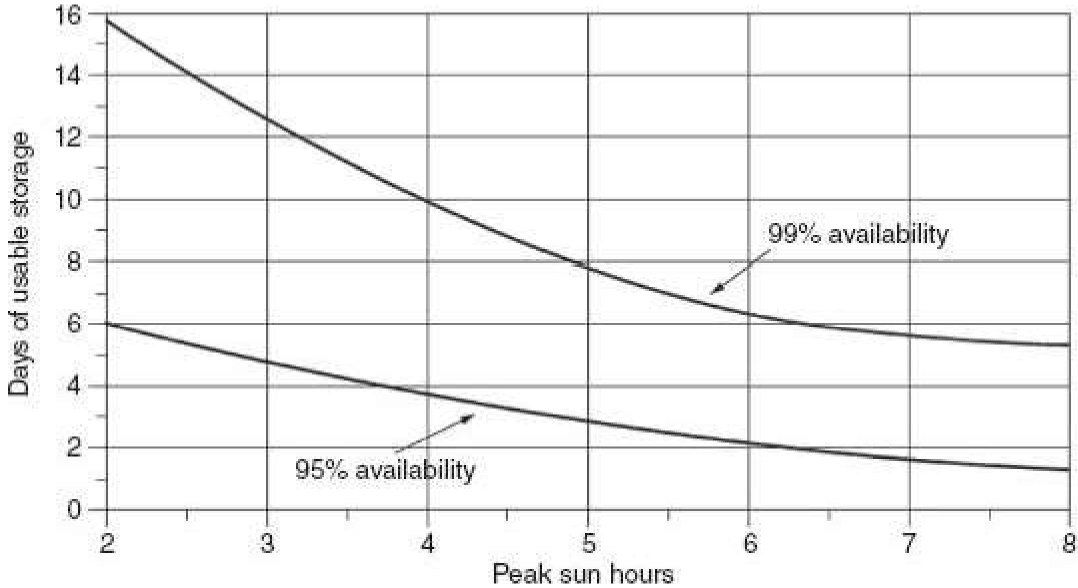
TABLE 6.13 Minimum System Voltages Based on Limiting Current to 100 A

Maximum AC Power	Minimum DC System Voltage
<1200 W	12 V
1200–2400 W	24 V
2400–4800 W	48 V

If good weather could always be counted on, battery sizing might mean simply providing enough storage to carry the load through the night and into the next day until the sun picks up the load once again. The usual case, of course, is one in which there are periods of time when little or no sunlight is available and the batteries might have to be relied on to carry the load for some number of days. During those periods, there may be some flexibility in the strategy to be taken. Some noncritical loads, for example, might be reduced or eliminated, and if a generator is part of the system, a tradeoff between battery storage and generator run times will be part of the design.

Given the statistical nature of weather and the variability of owner responses to inclement conditions, there are no set rules about how best to size battery storage. The key tradeoff will be cost. Sizing a storage system to meet the demand 99% of the time can easily cost triple that for one that meets demand only 95% of the time. As a starting point for estimating the number of days of storage to be provided, consider Figure 6.31, which is based on the excellent guidebook *Stand-Alone Photovoltaic Systems Handbook of Recommended Design Practices* (Sandia, 1995). The graph gives an estimate for days of battery storage needed to supply a load as a function of the peak sun hours per day in a given month. To account for a range of load criticality, two curves are given: one for loads that must be satisfied during 99% of the 8760 h in a year and one for less-critical loads, for which a 95% system availability is satisfactory.

FIGURE 6.31 Days of battery storage needed for a stand-alone system with 95% and 99% system availability. Peak sun hours are on a month-by-month basis. Based on Sandia Laboratories (1995).



The "usable" description of storage in Figure 6.31 assumes you have already accounted for impacts associated with maximum discharge levels and rates as well as temperature constraints (Figs. 6.26 and 6.28). The relationship between usable storage and nominal, rated storage (at C/20, 25°C) is given by

$$\text{Usable capacity} = \text{Nominal capacity } (C/20, 25°\text{C}) \times (\text{MDOD}) \times (\text{T, DR}) \quad (6.31)$$

where

MDOD = Maximum depth of discharge (default: 0.8 for lead–acid, deep discharge batteries, 0.25 for auto SLI; subject to freeze constraints given in Fig. 6.26).

TDR = Temperature and discharge-rate factor (Fig. 6.28)

The following example illustrates the process.

Example 6.12 Battery Sizing for the Off-Grid House in Boulder, CO. The household analyzed in Examples 6.9 and 6.10 has an expected AC demand of 6288 Wh/d. A decision has been made to size the batteries to provide a 95% system availability during an average month of the year. A backup generator will cover the other 5%. The batteries will be kept in a well-ventilated shed whose temperature may reach as low as −10°C. Make appropriate assumptions and design the battery bank.

Solution. We need to estimate both the average and peak loads that the batteries must supply. Since the batteries will deliver power through a charge controller and an inverter, we need to estimate its efficiency. At their peak, inverters have efficiencies in the 90+% range, but with significant fractions of less-than-peak loads, an overall efficiency of more like 85% is reasonable. Let us estimate the controller efficiency at 97%. The DC load is then

$$\text{Battery DC load} = \frac{\text{Household AC load}}{\text{Inverter efficiency} \times \text{Controller efficiency}} = \frac{6288 \text{ Wh/d}}{0.85 \times 0.97} = 7626 \text{ Wh/d} \quad (6.32)$$

To translate that into ampere-hours of battery capacity, we need to pick a system voltage. A glance at the load described in Example 6.9 shows if everything in the house was turned on at the same time the demand would be about 3.6 kW, which by Table 6.13 suggests we need a 48-V system voltage to keep the peak current below 100 A. With a 48-V system voltage the batteries need to supply

$$\text{Load} = \frac{7398 \text{ Wh/d}}{48 \text{ V}} = 159 \text{ Ah/d at } 48 \text{ V}$$

In Example 6.9, we decided to use an L + 15 tilt angle for which the average monthly insolation is 4.5 kWh/m^2/d (Appendix G), so from Figure 6.31 it looks like a 3-day storage capacity will cover our needs about 95% of the time. So, the usable storage we need is

$$\text{Usable storage} = 159 \text{ Ah/d} \times 3 \text{ days} = 477 \text{ Ah at } 48\text{V}$$

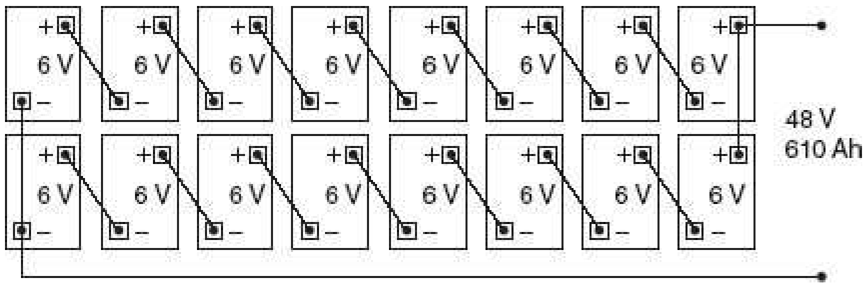
We will pick deep discharge lead–acid batteries that can be routinely discharged by 80%. But, we need to check to see whether that depth of discharge will expose the batteries to a potential freeze problem. From Figure 6.26, at −10°C the batteries could be discharged to over 95% without freezing the electrolyte, so an 80% discharge is acceptable.

Batteries that are nominally rated at C/20 and 25°C will be operated in much colder conditions which degrade storage capacity, but they will be discharged at a much slower rate, which increases capacity. Figure 6.28 suggests that at −10°C and a C/72 rate, which covers the 3 days of no sun that we have designed for, storage capacity would be about 0.97 times the nominal capacity.

Applying the 0.80 factor for maximum discharge and the 0.97 factor for discharge rate and temperature in Equation 6.33 gives

$$\text{Nominal (C/20, 25°C) capacity} = \frac{\text{Usable capacity}}{\text{MDOD} \times (\text{TDR})} = \frac{477 \text{ Ah}}{0.80 \times 0.97} = 615 \text{ Ah at } 48 \text{ V} \quad (6.33)$$

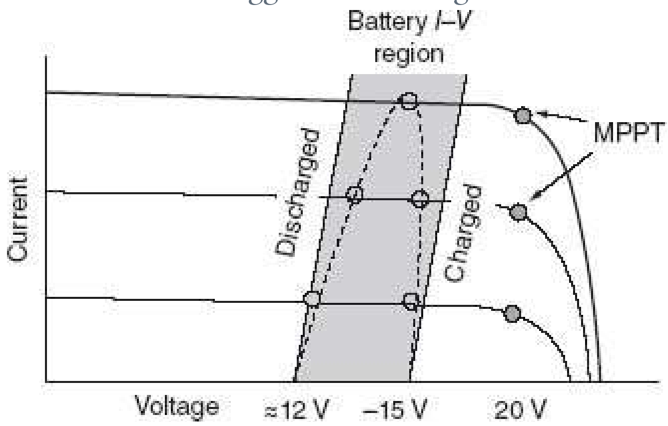
Table 6.12 can help us pick a battery. A single string of 12 of those 4-V, 546-Ah Rolls Surette batteries would be a bit undersized, but the redundancy advantage that goes with two strings suggests we use 16 of the 6-V, 305-Ah Concorde 3050T batteries as shown below.



6.5.10 Sizing an Array with No MPP Tracker

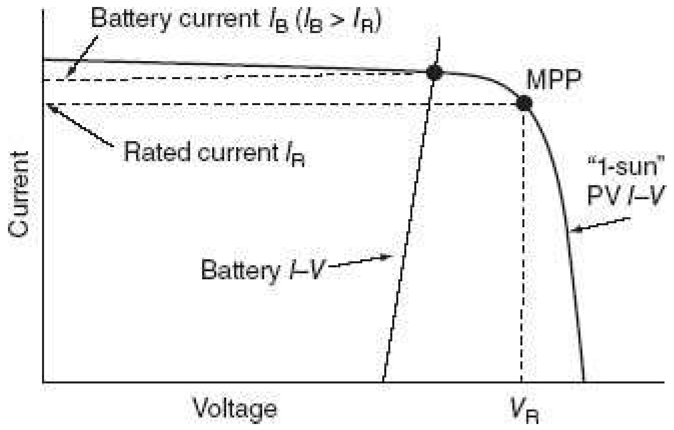
In Section 6.5.4, stand-alone system PV arrays were sized assuming a maximum power point tracker. By including an MPPT, we were able to use the simple "peak-hours" approach to system sizing. When there is no MPP tracker, the operating point is determined by the intersection of the *I*–*V* curve for the batteries with the *I*–*V* curve of the PVs. Figure 6.32 suggests the daily path of the operating point, which typically will be well below the knee where an MPPT would operate. Without an MPPT, the something like 20% or so of the potential charging will be lost.

FIGURE 6.32 With PVs directly connected to batteries, the operating point will move around within the shaded area as voltage rises during charging and as insolation changes throughout the day. The dotted line suggests a morning-to-afternoon trajectory.



One way to analyze the performance of PV-to-battery, direct coupling is suggested in Figure 6.33. As shown, during battery charging, the operating point is almost always above the knee of the PV *I*–*V* curve, which means that charging current will exceed the rated current of the PVs. It is a fairly conservative estimate, therefore, to simply use the rated current of the PVs as an indication of the battery charging current at 1-sun insolation. There are circumstances in which this assumption should be checked, as for example, when a 12-V battery is charged in a high temperature environment with a "self-regulating" PV module having fewer than the usual 36 cells in series. Fewer cells and higher temperatures move the MPP toward the battery *I*–*V* curve and the conservatism of the rated-current assumption decreases.

FIGURE 6.33 Estimating battery charging at 1-sun to be the rated current of the PVs is a fairly conservative assumption.



A simple sizing procedure is based on the same "peak-hours" approach used for grid-connected systems, except it will be applied to current rather than power. So, for example, an area with 6 kWh/m^2/d of insolation is treated as if it has 6 h/d of 1-sun irradiation. Then, using the rated current I_R at 1-sun times peak hours of sun gives us ampere-hours of current provided to the batteries.

The product of rated current I_R times peak hours of insolation provides a good starting-point estimate for ampere-hours delivered to the batteries. It is common practice to apply a derating factor of about 0.9 to account for dirt and gradual aging of the modules. The temperature and module-mismatch factors that were quite important for grid-connected systems with MPPTs are usually ignored for PV-battery systems since the operating point is so far from the knee those variations are minimal.

Another important aspect of this approach is that it is based on ampere-hours from the PVs into the batteries, and ampere-hours from the batteries to the load. That is, the appropriate battery efficiency measure will be the Coulomb efficiency (Ah_{out}/Ah_{in}) rather than the actual energy efficiency. The current delivered by batteries, to the controller, inverter, and load, is

(6.34)

$$\text{Ah from batteries} = I_R \times \text{Peak sun hours} \times \text{PV derate} \times \text{Coulomb efficiency}$$

Energy delivered through the controller and inverter to satisfy the household load is

$$\text{Watt-hours per day} = \text{Ah/d from batteries} \times \text{System voltage} \times \text{Controller } \eta \times \text{Inverter } \eta \quad (6.35)$$

Example 6.13 PVs for the Boulder House, Without an MPPT. The Boulder house in Example 6.10 needs 6.29 kWh/d of AC delivered from the inverter. We sized a 1.98-kW$_{DC}$ array to meet the load in an average month with 5.3 kWh/m^2/d insolation. In Example 6.12, we settled on a 48-V system voltage for the batteries.

Assuming a 90% Coulomb efficiency, 0.90 module derate, an 85%-efficient inverter, and a 97%-efficient controller, size the PV array using the peak-hours approach for a system without an MPPT. Pick a collector type from the list given in Table 5.3.

Solution. We will pick a collector based on the need for the array to supply a 48-V battery system. The best match for the 48-V system voltage would be the Yingli 245, with a rated voltage 30.2 V, so two of those in series would assure the knee is above the 48-V system voltage. All of the other collector options would result in even higher voltages at the knee, which would be excessively wasteful. The Yingli has a rated current of 8.11 A.

From Equations 6.34 and 6.35, the average energy delivered to the load by a series pair of these modules in one string would be

$$\begin{aligned}\text{Energy} &= 8.11\ \text{A} \times 48\ \text{V} \times 5.3\ \text{h/d} \times 0.90 \times 0.90 \times 0.85 \times 0.97 \\ &= 1378\,\text{Wh/d per string}\end{aligned}$$

The number of strings needed to deliver the 6.29-kWh/d load would be

$$\text{Parallel strings} = \frac{6.29\ \text{kWh/d}}{1.378\ \text{kWh/d}} = 4.6\ \text{strings of 2 modules each}$$

Since we sized for an average month, which already means a number of months will be underserved, let us round up to five strings. Energy delivered in the worst month of the year, December, with 4.5 h/d of insolation, the energy delivered would be,

$$\begin{aligned}\text{Energy} &= 5\ \text{strings} \times 8.11\ \text{A} \times 48\ \text{V} \times 4.5\ \text{h/d} \times 0.9 \times 0.9 \times 0.85 \times 0.97 \\ &= 5850\ \text{Wh/d}\end{aligned}$$

which is 93% of the 6.29-kWh/d load in December.

The rated power of the array will be

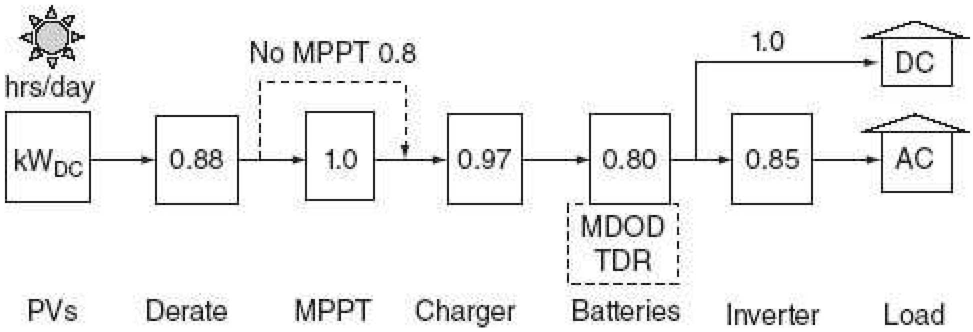
$$P_R = 5\ \text{strings} \times 2\ \text{modules/string} \times 245\ \text{W/module} = 2.45\ \text{kW}_{\text{DC}}$$

In the above example, the design calls for a 2.45-kW array to meet the average load without an MPPT. Using similar design criteria in Example 6.10, we found that a 1.98-kW array would deliver comparable performance with an MPPT. That is, the PV system without an MPPT needs about 20% more PVs than the system with an MPPT. We will use that as a guideline for a 0.80 penalty factor for non-MPPT systems. Doing so will let us create a simple spreadsheet approach that covers both types of off-grid system without having to develop separate procedures.

6.5.11 A Simple Design Template

With all of the accumulated efficiency factors presented thus far, it might help to summarize them in a figure and then incorporate them into a relatively straightforward spreadsheet. Begin with Figure 6.34 in which the flow of power is shown through all of the processes involved in supplying power to a combination of DC and AC loads.

FIGURE 6.34 Power flows in an off-grid PV system. Values are suggested as “typical” parameter estimates.



The suggested parameter values in Figure 6.34 are meant to be "typical" estimates. We will step through each one:

PVs	This analysis is based on a "peak-hours" approach in which all the following factors affect the final energy delivered to both AC and DC loads.
Derate	This 0.88 derate factor is meant to account for dirt, module mismatch, wiring losses, and so on, but not inverter efficiency.
MPPT	If there is an MPPT this factor is 1.0. If not, a value of 0.80 is suggested to account for the operating point of a non-MPPT battery, PV system being well to the left of the knee of the PV *I–V* curve (see Example 6.13).
Charger	This factor accounts for the modest losses in the charge controller.
Batteries	The 0.80 factor is an estimate of the round-trip efficiency of putting energy into a battery and getting it back out again. To the extent that daytime generation goes straight from PVs to the load, bypassing the batteries, this factor could be increased.
MDOD, TDR	The maximum depth of discharge (MDOD) and the TDR are not loss factors, but they are needed to size the battery pack.
Inverter	The 0.85 efficiency is lower than that for a grid-connected system, since much of the time it will be operating well below its optimum point. For DC loads, there is no inverter, so a factor of 1.0 is used.

Table 6.14 presents a spreadsheet approach that incorporates the key design decisions.

TABLE 6.14 An Example Design Template for Off-Grid Systems (Illustrating a System Without an MPPT Including Both AC and DC Loads)

HOUSEHOLD LOADS					
AC Loads	No	W ea.	Watts	h/d	Wh/d
Refrigerator, 19 cu ft	1	300	300		1080
Lights (6 at 25 W, 6 h/d)	6	25	150	6	900
LCD TV 3 h/d (on)	1	200	200	3	600
LCD TV 21 h/d (standby)	1	2	2	21	42
Satellite with DVR	1	3	44	3	132
Satellite (standby)	1	21	43	21	903
Assorted electronics at 3 W ea	10	3	30	24	720
Microwave at 12 min/d	1	1200	1200	0.2	240
Range burner (small)	1	1200	1200	1	1200
Clothes washer (0.25 kW; load/wk at 0.3 kWh)	4	250	250		171
Laptop computer (2 h/d at 30 W)	1	30	30	2	60
Well pump (120 gal/d at 1.5 gal/min)	1	180	180	1.33	240
Other	0		0		0
Total			3566		6288
DC Loads					
Circular saw	1	900	900	0.5	450
Other	0		0		0
Total			900		450

BATTERY DATA		
Days of storage	3.0	Figure 6.31 may help
System voltage	48	Typical 12, 24, 48 V
Maximum current (A)	93	With all loads on at once; try to keep below 100A
Inverter efficiency η	85%	Typical 85%
Battery DC delivery (Ah/d)	9.4	Ah/d = DC Wh/d / System voltage
Battery AC delivery (Ah/d)	154.1	Ah/d = AC Wh/d / System voltage / Inverter efficiency
Battery total delivery (Ah/d)	163.5	Ah/d = DC + AC
Maximum depth of discharge (MDOD)	80%	Typical 80%
Temperature and discharge rate (TDR) adjustment	97%	Figure 6.28
Minimum battery storage (Ah)	652	Ah = (dc+ac Ah/d) × (days) / (MDOD, TDR) / contr eff

PHOTOVOLTAIC DATA		
Design month insolation ($kWh/m^2/d$ = h/d)	5.3	Annual average is reasonable with backup generator
Derate (dirt, mismatch, wiring)	0.88	Typical 0.75; does not include battery losses
MPP impact	0.80	Typical 1.0 with MPPT; 0.8 without MPPT
Charge controller efficiency	97%	Typical 97%
Battery round-trip efficiency	80%	Typical 80%
$P_{DC,STC}$ (kW)	2.71	P_{DC} = Ah/d × V/(h/d × derate × MPP × controller × battery efficiency)

RESULTS	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec	Average
Insolation (h/d)	4.8	5.3	5.6	5.6	5.2	5.3	5.5	5.8	5.8	5.7	4.8	4.5	5.3
Load (kWh/d)	6.74	6.74	6.74	6.74	6.74	6.74	6.74	6.74	6.74	6.74	6.74	6.74	6.74
PV supply (kWh/d)	6.10	6.74	7.12	7.12	6.61	6.74	6.99	7.37	7.37	7.25	6.10	5.72	6.77
%Load (100% max)	91%	100%	100%	100%	98%	100%	100%	100%	100%	100%	91%	85%	97%

It should be easy to reverse-engineer Table 6.14 to create your own spreadsheet. The only modestly tricky part is calculating the month-by-month energy delivered by the PV system once it has been sized. For example, the energy delivered in August without an MPPT by a 2.71-kW_{DC} array exposed to 5.8 $kWh/m^2/d$ of insolation will be

$$\begin{aligned}\text{Energy to batteries} &= 2.71\ \text{kW} \times 5.8\ \text{h/d} \times 0.88\ (\text{derate}) \times 0.80\ (\text{no MPPT}) \\ &\quad \times 0.97\ (\text{controller}) \\ &= 10.73\ \text{kWh/d}\end{aligned}$$

We assume all of that goes through the 80%-efficient batteries before being delivered to the DC and AC loads. With 5.75% of the energy going to DC loads (9.4/163.5 Ah) with no inverter

$$\text{Energy delivered to DC loads} = 10.73\ \text{kWh/d} \times 0.80 \times 5.75\% = 0.49\ \text{kWh/d}$$

The other 94.25% of the power goes through the 85%-efficient inverter for AC loads:

$$\text{Energy to AC loads} = 10.73\ \text{kWh/d} \times 0.80 \times 94.25\% \times 0.85\ (\text{inv})$$
$$= 6.88\ \text{kWh/d}$$

Total energy delivered to loads = 0.49 + 6.88 = 7.37 kWh/d, which is more than was needed in the design, so for that month the percent load is pegged at 100% under the assumption that carryover of extra energy from 1 month to another is unlikely.

6.5.12 Stand-alone PV System Costs

A PV system designed to supply the entire load in the worst season usually delivers much more energy than is needed during the rest of the year. Outside of the tropics, it is not at all unusual for the energy supplied during the best month to be as much as double that of the worst month. After estimating the cost of a system that has been designed to be completely solar, a buyer may very well decide that a hybrid system with most of the load covered by PVs and the remainder supplied by a generator is worth considering. The key to that decision is estimating the relationship between shrinking the PV system and increasing the fraction of the load carried by the generator.

When a generator is included in the system, it is most convenient if is made to be an inverter-charger. That is, one that converts DC from the batteries into AC for the load, as well as converting AC from the generator into DC to charge the batteries. Switching from one mode to the other can be done manually or with an automatic transfer switch in the unit itself. The generator can be sized just to charge the batteries, which is the usual case, or it can be sized large enough to charge batteries and simultaneously run the entire household.

With a hybrid system, the battery storage bank can be smaller since the generator can charge the batteries during prolonged periods of poor weather. One constraint on how small storage can be is to check to be sure that the load cannot discharge the batteries at too fast a rate—certainly no faster than C/5. A nominal 3-day storage system is often recommended since it will avoid discharging too rapidly, while at the same time keeping the number of times the generator has to be fired up to a reasonable level. Finally, the generator should be sized so that it does not charge the batteries too rapidly—again, certainly no faster than C/5.

Generators are somewhat costly, depending on the quality of the machine. They require periodic oil changes, tune-ups, and major overhauls. Home-size generators burn fuel at varying rates and are especially dependent on the fraction of full load at which they operate.

Example 6.14 Fuel Cost for a Diesel Generator. Manufacturer's specifications for a 9.1-kW diesel generator indicate fuel use at full rated power is 4 L/h (1.06 gal/h), while at 50% load it consumes 2.5 L/h (0.66 gal/h). If diesel costs $4/gal ($1.057/L), find the fuel cost per kWh of generation at full and half load.

Solution. The energy efficiency and fuel cost at 100% of rated power and at 50% of rated power will be

$$\text{kWh/gal: Rate (100\% load)} = \frac{9.1\ \text{kW}}{1.057\ \text{gal/h}} = 8.61\ \text{kWh/gal}$$

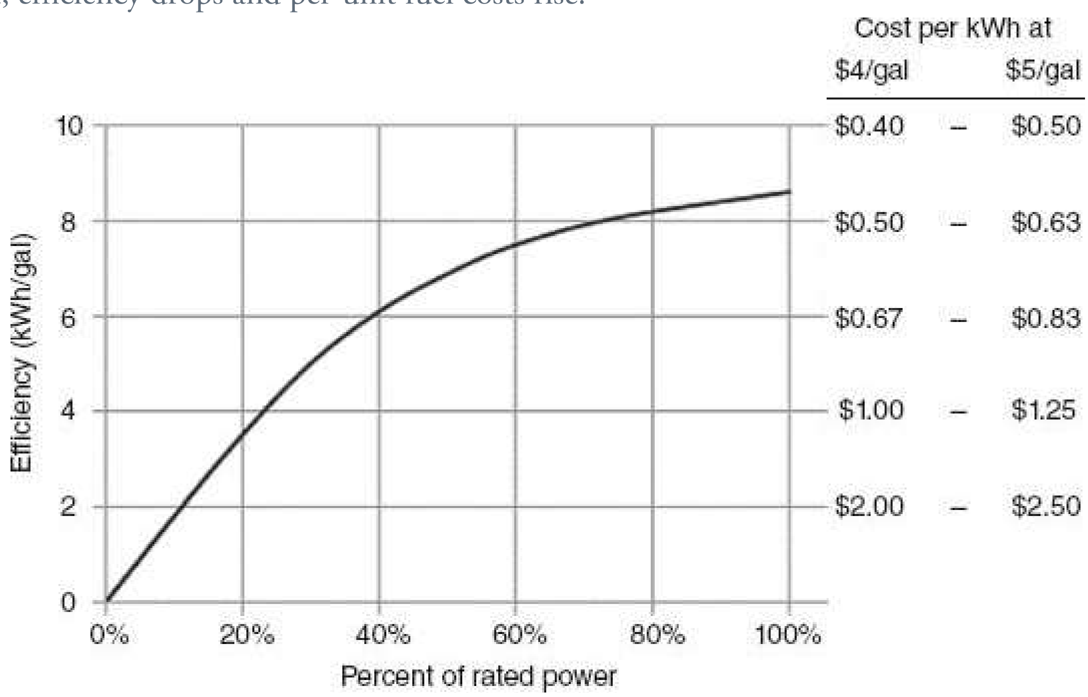
$$\text{Rate (50\% load)} = \frac{9.1 \times 0.5\ \text{kW}}{0.66\ \text{gal/h}} = 6.89\ \text{kWh/gal}$$

$$\text{At \$4/gallon: Cost (100\% load)} = \frac{\$4/\text{gal}}{8.61\ \text{gal/h}} = \$0.46/\text{kWh}$$

$$\text{: Cost (50\% load)} = \frac{\$4/\text{gal}}{6.89\ \text{gal/h}} = \$0.58/\text{kWh}$$

Example 6.14 provides a starting point for estimating the cost of electricity from a standby diesel generator. The full power curve for this particular generator is shown in Figure 6.35, with cost per kWh estimates at fuel costs of \$4/gal and \$5/gal. As shown, the unit cost is very dependent on the percentage of rated power actually being generated, which points out the importance of proper sizing. As rough guidelines, backup generators produce roughly 5–10 kWh/gal of fuel, which at \$4/gal works out to about \$0.40–\$0.80/kWh.

FIGURE 6.35 Efficiency curve for a 9.1-kW diesel generator. When run at only partial load, efficiency drops and per-unit fuel costs rise.



Given the wide range of uncertainties that accompany the economics of off-grid PV systems, it is difficult to generalize their cost-effectiveness. They may be located a few miles from a major economic center with easy access to sources of quality materials and well-trained labor, or they may be tens or hundreds of miles from such amenities. But despite these difficulties, it may be worthwhile to at least step through an example with some approximate estimates of costs.

Example 6.15 A Rough Cost Analysis for an Off-Grid System. Using the following rough unit-cost estimates, find the cost of electricity for the stand-alone system designed in Table 6.14 (2.71 kWp, 632-Ah battery, 6.77 kWh/d delivered). Assume financing with a 4%, 20-year loan.

PV array cost	$2/Wp
Battery cost	$150/kWh
BOS hardware	$2/Wp
BOS nonhardware	30% of hardware costs

Solution. Using system data from Table 6.14

$$\text{PV array cost} = 2.71\ \text{kWp} \times 1000\ \text{W/kW} \times \$2/\text{Wp} = \$5420$$

$$\text{Battery cost} = \frac{632\ \text{Ah} \times 48\ \text{V}}{1000\ \text{VAh/kWh}} \times \$150/\text{kWh} = \$4550$$

$$\text{BOS hardware cost} = 2710\ \text{Wp} \times \$2/\text{Wp} = \$5420$$

$$\text{BOS nonhardware} = 30\%\ (\$5420 + \$4550 + \$5420) = \$4617$$

$$\text{Total wholesale} = \$5420 + \$4550 + \$5420 + \$4617 = \$20{,}007$$

$$\text{Amortized with a CRF}\ (4\%,\ 20\ \text{years}) = 0.0736/\text{yr}\ (\text{Table 6.4})$$

$$\text{Cost per kWh} = \frac{\$20{,}007 \times 0.0736/\text{yr}}{6.77\ \text{kWh/d} \times 365\ \text{d/yr}} = \$0.60/\text{kWh}$$

The above analysis, without any incentives, resulted in 60 ¢/kWh PV electricity, which makes it potentially financially competitive with diesel. And, the PV system is certainly a lot cleaner and more user-friendly than diesel.

6.6 PV-POWERED WATER PUMPING

One of the most economically viable PV applications today is water pumping in remote areas. For an off-grid home, a simple PV system can raise water from a well or spring and store it in either a pressurized or an unpressurized tank for domestic use. Water for irrigation, cattle watering, or village water supplies—especially in developing countries—can be critically important and the value of a PV water-pumping system in these circumstances can far exceed its costs.

The simplest PV water-pumping systems consist of just a PV array directly connected to a DC pump. Water that is pumped when the sun is out may be used during those times or it can be stored in a tank for later use. The costs and complexities of batteries, controllers, and inverters can be eliminated, resulting in a system that combines simplicity, low cost, and reliability. On the other hand, matching PVs and pumps in such directly coupled systems (without battery storage) and predicting their daily performance is actually a quite challenging task.

As suggested in Figure 6.36, a simple, directly coupled PV-pump system has an electrical side in which PVs create a voltage *V* that drives current *I* through wires to a motor load, and a hydraulic side in which a pump creates a pressure, H (for *head*) that drives water at some flow rate Q through pipes to some destination. The figure suggests the hydraulic side is a *closed* loop with water circulating back to the pump, but it may also be an *open* system in which water is raised from one level to the next and then released. On the electrical side of the system, the voltage and current delivered at any instant are determined by the intersection of the PV I–V curve and the motor I–V curve. In the hydraulic system, H is analogous to voltage while Q is analogous to current. As shown in Figure 6.37, the intersection between the H–Q curve drawn for the pump itself and the H–Q pressure and flow curve for the load determines the hydraulic operating point. The analogies are also true with regard to power. For both sides of the system, maximum power is delivered to the load at an operating point located on the knee of their respective system curves.

FIGURE 6.36 The electrical characteristics of the PV–motor combination need to be matched to the hydraulic characteristics of the pump and its load.

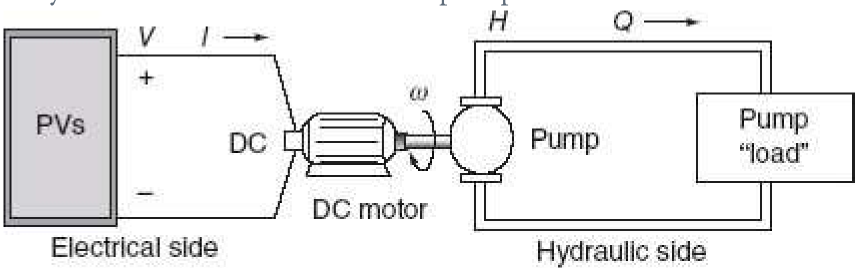
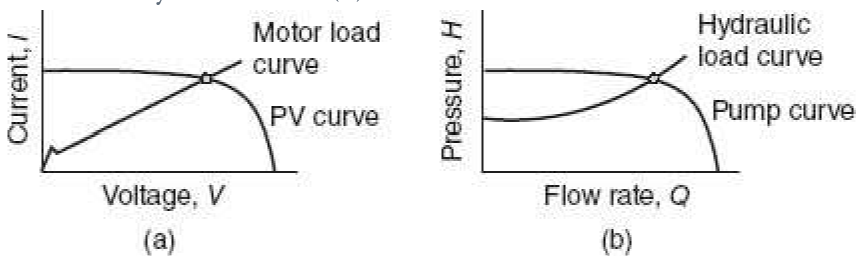


FIGURE 6.37 Showing the analogies between the I–V curves on the electrical side (a) and the H–Q curves on the hydraulic side (b).



While the system in Figure 6.36 could not be simpler, the analysis of how it will operate is rather tricky. The power delivered to the pump motor will vary throughout the day as insolation changes, which means the pump curve on the hydraulic side will vary up and down as well. With both operating points moving around, predicting the amount of water pumped over a day's time becomes a challenge.

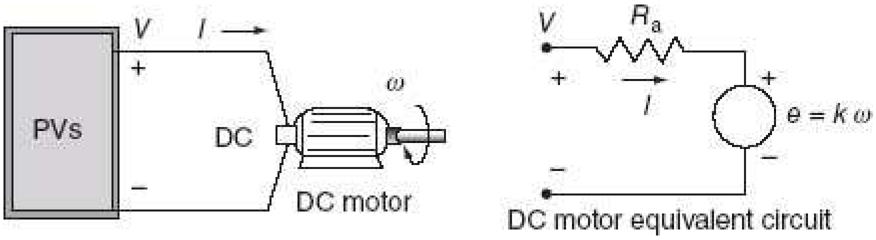
6.6.1 The Electrical Side of the System

Since the electrical system in question is DC, the pump will usually be driven by a DC motor. Most are permanent magnet DC motors, which can be modeled as shown in Figure 6.38. Note as the motor spins, it develops a *back electromotive force* (emf) e, which is a voltage proportional to the speed of the motor (ω) that opposes the voltage supplied by the PVs. From the equivalent circuit, the voltage–current relationship for the DC motor is simply

(6.36) $$V = IR_a + e = IR_a + k\omega$$

where back emf $e = k\omega$ and R_a is the armature resistance.

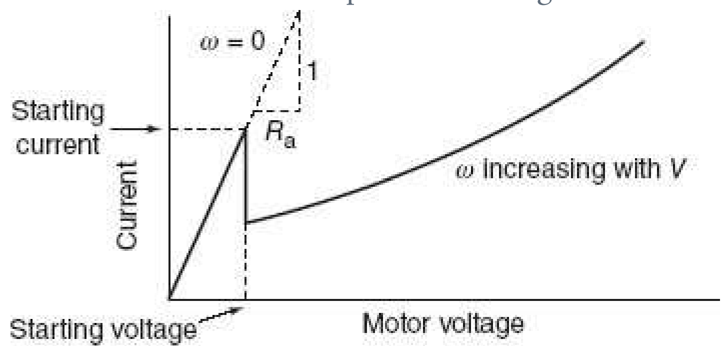
FIGURE 6.38 Electrical model of a permanent magnet DC motor.



A DC motor runs at nearly constant speed for any given applied voltage even though the torque requirement of its load may change. For example, as the torque requirement increases, the motor slows slightly, which drops the back emf and allows more armature current to flow. Since motor torque is proportional to armature current, the slowing motor draws more current, delivers more torque to the load, and regains almost all of its lost speed.

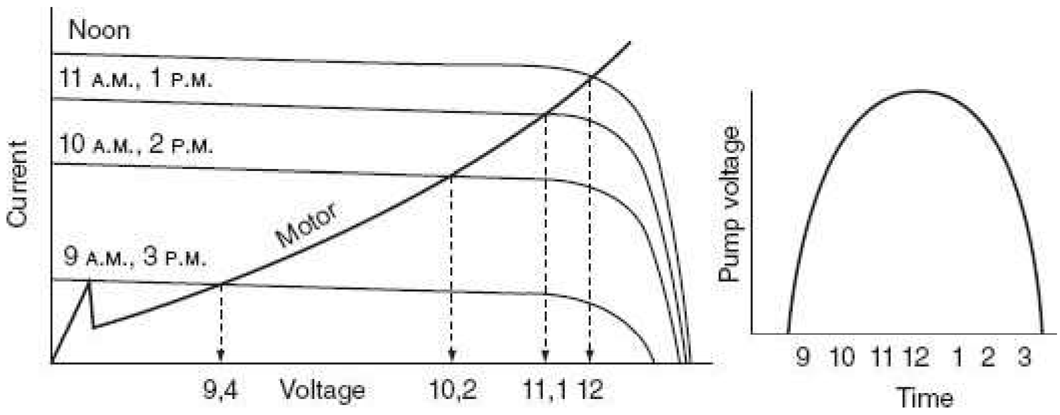
Based on Equation 6.36, the electrical characteristic curve of a DC motor will appear to be something like the one shown in Figure 6.39. Note that at start-up, while $\omega = 0$, the current rises rapidly with increasing voltage until current is sufficient to create enough starting torque to break the motor free from static friction. Once the motor starts to spin, back emf drops the current, and thereafter, I rises more slowly with increasing voltage. Note that if you stall a DC motor while the voltage is way above the starting voltage, the current may be so high that the armature windings can burn out. That is why you should never leave power on a DC motor if the armature is mechanically stuck for some reason.

FIGURE 6.39 Electrical characteristics of a permanent magnet DC motor.



A DC motor I–V curve is superimposed onto a set of hour-by-hour PV I–V curves in Figure 6.40. The mismatch of operating points with the ideal MPP is apparent. Note in this example that the motor does not start until about 9:00 A.M., when insolation and current are finally high enough to overcome static friction. This incurs some morning losses, which could be eliminated with a DC-to-DC converter that would transform the low, morning PV current into high enough current to start the motor sooner. But, that makes the system more complicated than the simple one being described here.

FIGURE 6.40 Superimposing the DC motor curve on hourly PV I–V curves yields hour-by-hour voltages supplied to the pump motor.



6.6.2 Hydraulic Pump Curves

Pumps suitable for PV-powered systems generally fall into one of two categories: *centrifugal* pumps and *positive displacement* pumps. Centrifugal pumps have fast-spinning impellers that literally throw the water out of the pump, creating suction on the input side of the pump, and pressure on the delivery side. When these are installed above the water, they are limited by the ability of atmospheric pressure to push water up into the suction side of the pump—that is, to a theoretical maximum of about 32 ft; in practice, that is more like 20 ft. When the pump is installed below the water line, however, the pump can push water up hundreds of feet. Submersible pumps with waterproof housings for the motor are suspended in a well using the same pipe that the water is pumped through. In this configuration, centrifugal pumps can push water over 1000 vertical feet. One of the disadvantages of centrifugal pumps, however, is that their speedy impellers are susceptible to abrasion and clogging by grit in the water. When powered by PVs, they are also particularly sensitive to changes in solar intensity during the day.

Positive displacement pumps come in several types, including *helical pumps*, which use a rotating shaft to push water up a cavity; *jack pumps*, which have an above-ground oscillating arm that pulls a long drive shaft up and down (like the classic oil-rig pumper); and *diaphragm pumps* that use a rotating cam to open and close valves. The traditional hand pump and wind-powered water pumps are versions of jack pumps. Jack pumps use simple flap valves that work very much like hydraulic diodes. During each upward stroke of the shaft, a flap closes and a gulp of water is carried upward; during the downward stroke, the valve opens and new water enters a chamber to be carried upward on the next stroke. In general, positive displacement pumps pump at slower rates; so they are most useful in low volume applications. They easily handle high heads, however, and they are much less susceptible to gritty water problems than centrifugal pumps. They also are less sensitive to changes in solar intensity. A brief comparison of the two types of pumps is presented in Table 6.15.

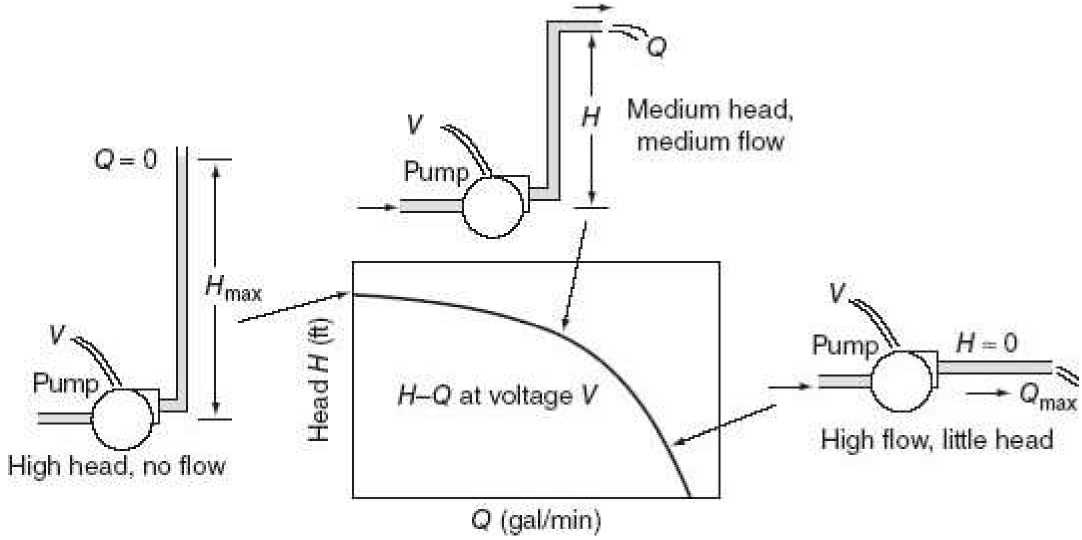
TABLE 6.15 A Comparison Between Centrifugal and Positive Displacement Pumps

Centrifugal	Positive Displacement
High speed impellers	Volumetric movement
Large flow rates	Lower flow rates
Loss of flow with higher heads	Flow rate less affected by head
Low irradiance reduces ability to achieve head	Low irradiance has little effect on head

Potential grit abrasion	Unaffected by grit

The graphical relationship between head and flow is called the *hydraulic pump curve*, illustrated in Figure 6.41. To help understand the shape of the curve, imagine a small centrifugal pump connected to a hose that has one end submerged in a pond. Raising the open end of the hose higher and higher (increasing the head) will result in less and less flow until a point is reached at which there is no flow at all. Similarly, as the open end is lowered, head decreases and flow increases until the hose is flat on the ground and the flow reaches a maximum.

FIGURE 6.41 Interpreting *H–Q* pump curves using a simple garden hose analogy.



Electrical *I–V* curves and hydraulic *Q–H* curves share many similar features. For example, recall that the electrical power delivered by a PV is the product of *I* times *V* and the MPP is at the knee of the *I–V* curve. For the hydraulic side, the power delivered by the pump to the fluid is given by

$$P = \rho HQ \tag{6.37}$$

where ρ is fluid density. In American units,

$$P(\mathrm{W}) = 8.34\ \mathrm{lb/gal} \times H(\mathrm{ft}) \times Q(\mathrm{gal/min}) \times (1\ \mathrm{min}/60\ \mathrm{s}) \times 1.356\ \mathrm{W/(ft\text{-}lb/s)}$$

$$P(\mathrm{W}) = 0.1885 \times H(\mathrm{ft}) \times Q(\mathrm{gal/min}) \tag{6.38}$$

In SI units,

$$P(\mathrm{W}) = 9.81 \times H(\mathrm{m}) \times Q(\mathrm{L/s}) \tag{6.39}$$

When *Q* is zero, there is no power delivered to the fluid; when the head *H* is zero, there is no power delivered either. Similar to PV curves, the MPP occurs at the spot at which you can fit the biggest rectangle under the *H–Q* curve.

Note the pump curve suggested in Figure 6.41 is a single curve corresponding to a particular voltage applied to the pump. As we have seen, in a directly coupled PV-to-pump system, pump voltage varies throughout the day (Fig. 6.40). Many manufacturers of pumps intended for solar applications will supply pump curves for voltages corresponding to nominal 12-V module voltages. Figure 6.42 shows an example of a set of pump curves for the Jacuzzi SJ1C11 DC centrifugal pump, which is intended for use with PVs. Individual curves have been given for 15-, 30-, 45-, and 60-V inputs. A typical "12-V" PV module operating near the knee of its *I*–*V* curve delivers about 15 V, so these pump voltages are meant to correspond to 1, 2, 3, and 4, typical "12-V" PV modules wired in series. Also shown are indications of the efficiency of the pump as a function of flow rate and head. Note that the peak in efficiency (about 44%) occurs along the knee of the pump curves, which is exactly analogous to the case for a PV *I*–*V* curve.

FIGURE 6.42 Pump curves for the Jacuzzi SJ1C11 pump for various input voltages. Pump efficiencies are also shown, with the peak along the knee of the curves.

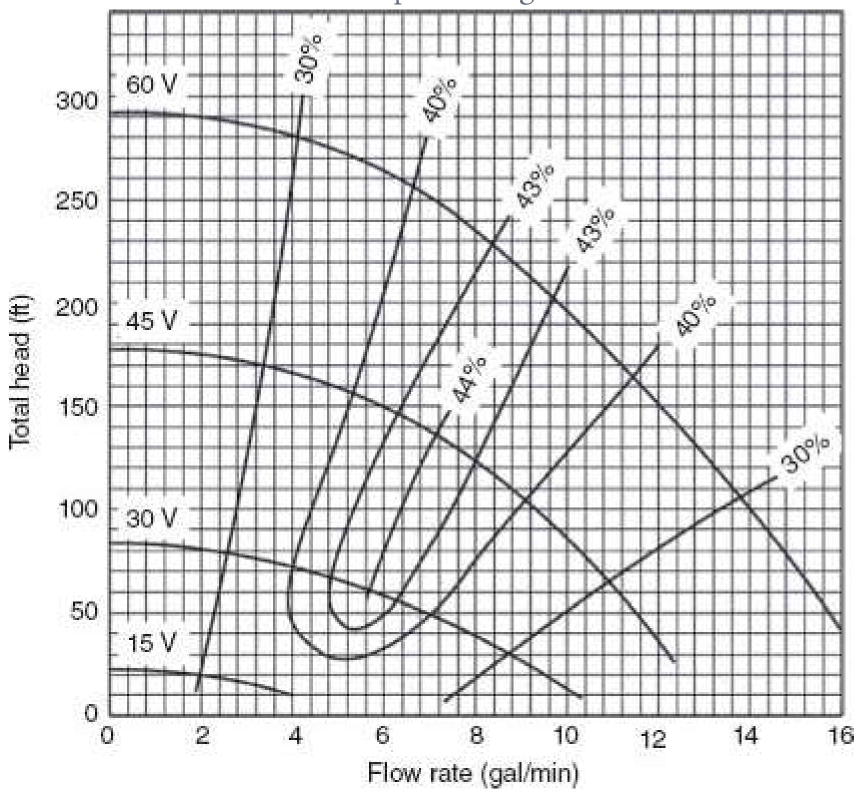


Figure 6.42 has enough information on it to derive the pump *I*–*V* curve. By selecting various intersection points where pump efficiency and pump voltage lines cross and coupling those data with Equation 6.38, we can create a table of pairs of *I* and *V*. For example, the 30%-efficiency line crosses the 15-V line at Q = 2 gal/min and H = 20 ft. From Equation 6.38

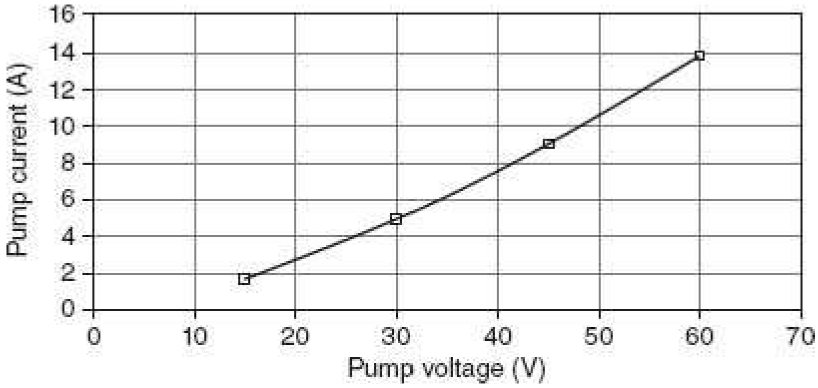
$$P = \frac{0.1885 \times H(\text{ft}) \times Q(\text{gal/min})}{\text{Efficiency}} = \frac{0.1885 \times 20 \times 2}{0.30} = 25 \text{ W}$$

And, from $P = VI$, at 15 V, current is 25 W/15 V = 1.7 A. Table 6.16 shows the data points used to plot the pump *I*–*V* curve shown in Figure 6.43.

TABLE 6.16 Deriving the Pump *I*–*V* Curve Using Data From Figure 6.42

V	Gal/min	Head (ft)	Efficiency (%)	P (W)	I (A)
15	2	20	30	25	1.7
30	5.6	62	44	149	5.0
45	6.4	145	43	407	9.0
60	6.8	258	40	827	13.8

FIGURE 6.43 The pump *I–V* curve derived from Figure 6.42.



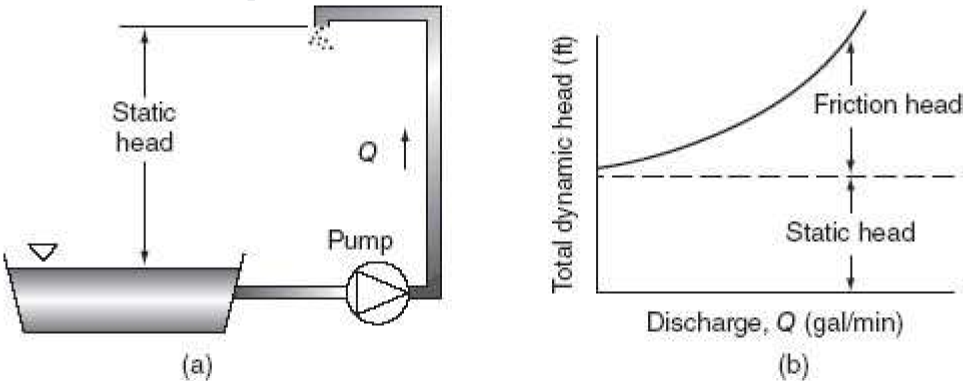
If the *I–V* pump curve just derived in Figure 6.43 is superimposed onto the *I–V* curve for PVs directly connected to the pump, the intersection point determines the voltage delivered to the pump. To determine fluid flow rates, however, we need to introduce the hydraulic side of the system.

6.6.3 Hydraulic System Curves

Figure 6.44 shows an open system in which water is to be raised from one level to the next. The vertical distance between the lower water surface and the elevation of the discharge point is referred to as the *static head* (or gravity head), and in the United States, it is usually given in "feet or inches of water." Head can also be measured in units of pressure, such as pounds per square inch (psi) or Pascals (1 psi = 6895 Pa). To convert between these two equivalent approaches to units, just picture the pressure that a cube of water exerts on its base. For example, a 1-ft cube weighing 62.4 lb would exert a pressure on its 144 square inches of base equal to

$$1 \text{ ft of head} = 62.4 \text{ lb}/144 \text{ in}^2 = 0.433 \text{ psi} \quad (6.40)$$

FIGURE 6.44 An "open" system (a) and the resulting "system curve" (b) showing the static and friction head components.



Conversely, 1 psi = 2.31 ft of water. Typical city water pressure is about 60 psi which corresponds to a column of water roughly 140 ft high.

If the pump is capable of supplying only enough pressure to the column of water to overcome the static head, the water would rise in the pipe and just make it to the discharge point and then stop. In order to create flow, the pump must provide an extra amount of head to overcome friction losses in the piping system. These friction losses rise roughly as the square of the flow velocity (as is suggested in Fig. 6.44). They depend on the roughness of the inside of the pipe and the numbers and types of bends and valves in the system. For example, the pressure drop per 100 ft of plastic water pipe for various flow rates and diameters is presented in Table 6.17. In keeping with U.S. tradition, flow rates are given in gal/min, pipe diameters in inches, and head in feet of water.

TABLE 6.17 Pressure Loss Due to Friction in Plastic Pipe for Various Nominal Tube Diameters[a]

Gal/min	0.5 in	0.75 in	1 in	1.5 in	2 in	3 in
1	1.4	0.4	0.1	0.0	0.0	0.0
2	4.8	1.2	0.4	0.0	0.0	0.0
3	10.0	2.5	0.8	0.1	0.0	0.0
4	17.1	4.2	1.3	0.2	0.0	0.0
5	25.8	6.3	1.9	0.2	0.0	0.0
6	36.3	8.8	2.7	0.3	0.1	0.0
8	63.7	15.2	4.6	0.6	0.2	0.0
10	97.5	26.0	6.9	0.8	0.3	0.0
15		49.7	14.6	1.7	0.5	0.0
20		86.9	25.1	2.9	0.9	0.1

[a]Units are feet of water per 100 ft of tube.

Table 6.18 gives pressure drops of various plumbing fittings expressed as equivalent lengths of pipe. For example, each 2-in 90° elbow (ell) in a plumbing run adds to the pressure drop the same amount as would 5.5 ft of straight pipe. So we can add up all the bends and valves in a pipe run and find what equivalent length of straight pipe would have the same pressure drop.

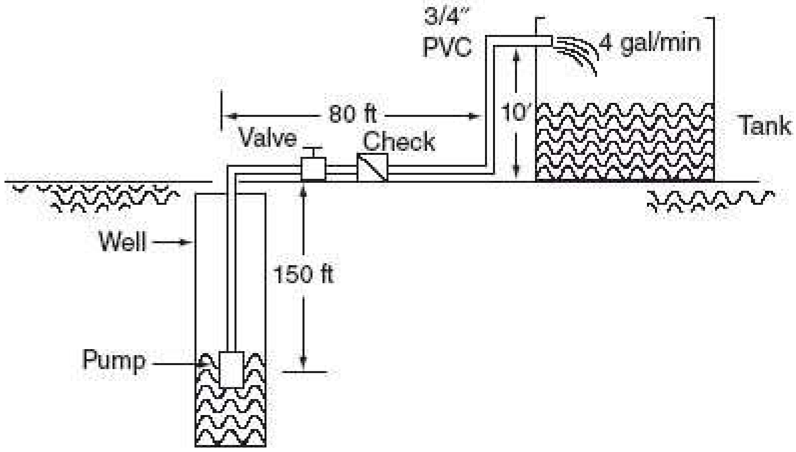
TABLE 6.18 Friction Loss in Valves and Elbows Expressed as Equivalent Lengths of Tube[a]

Fitting	0.5 in	0.75 in	1 in	1.5 in	2 in	3 in
90° ell	1.5	2.0	2.7	4.3	5.5	8.0
45° ell	0.8	1.0	1.3	2.0	2.5	3.8
Long sweep ell	1.0	1.4	1.7	2.7	3.5	5.2
Close return bend	3.6	5.0	6.0	10.0	13.0	18.0
Tee-straight run	1.0	2.0	2.0	3.0	4.0	5.0
Tee-side inlet or outlet	3.3	4.5	5.7	9.0	12.0	17.0
Globe valve, open	17.0	22.0	27.0	43.0	55.0	82.0
Gate valve, open	0.4	0.5	0.6	1.0	1.2	1.7
Check valve: swing	4.0	5.0	7.0	11.0	13.0	20.0

[a]Units are feet of pipe for various nominal pipe diameters.

The sum of the friction head and the static head is known as the *total dynamic head* (H).

Example 6.16 Total Dynamic Head for a Well. What pumping head would be required to deliver 4 gal/min from a depth of 150 ft. The well is 80 ft from the storage tank and the delivery pipe rises another 10 ft. The piping is 3/4-in diameter PVC, there are three 90° elbows, one swing-type check valve, and one gate valve in the line.



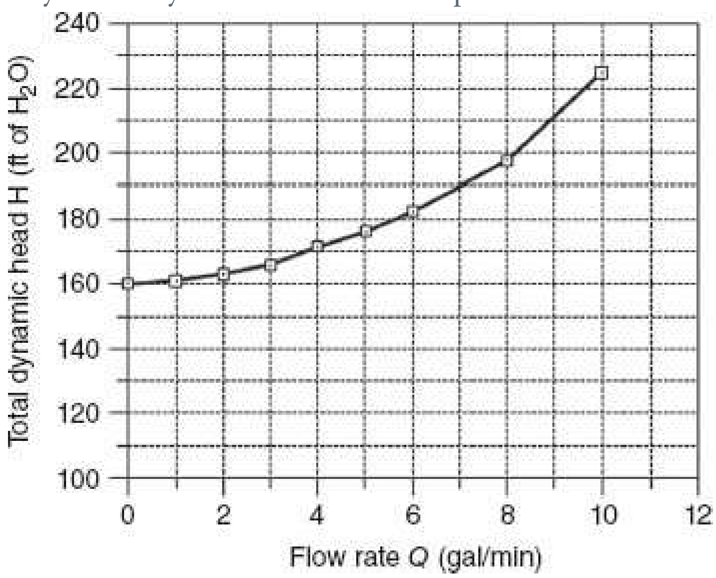
Solution. The total length of pipe is 150 + 80 + 10 = 240 ft. From Table 6.18, the three ells add the equivalent of 3 × 2.0 = 6 ft of pipe; the check valve adds the equivalent of 5 ft of pipe; the gate valve (assuming it is totally open) adds the equivalent of 0.5 ft of pipe. The total equivalent length of pipe is therefore 240 + 6 + 5+ 0.5 = 251.5 ft.

From Table 6.17, 100 ft of 3/4-in pipe at 4 gal/min has a pressure drop of 4.2 ft/100 ft of tube. Our friction-head requirement is therefore 4.2 × 251.5/100 = 10.5 ft of water.

The water must be lifted 150 + 10 = 160 ft (static head). Total head requirement is the sum of static and friction heads, or 160 + 10.5 = 170.5 ft of water pressure.

If the process followed in Example 6.16 is repeated for varying flow rates, a plot of total dynamic head *H* (static plus friction) versus flow rate, called the *hydraulic system curve*, can be derived. The hydraulic system curve for the above example is given in Figure 6.45.

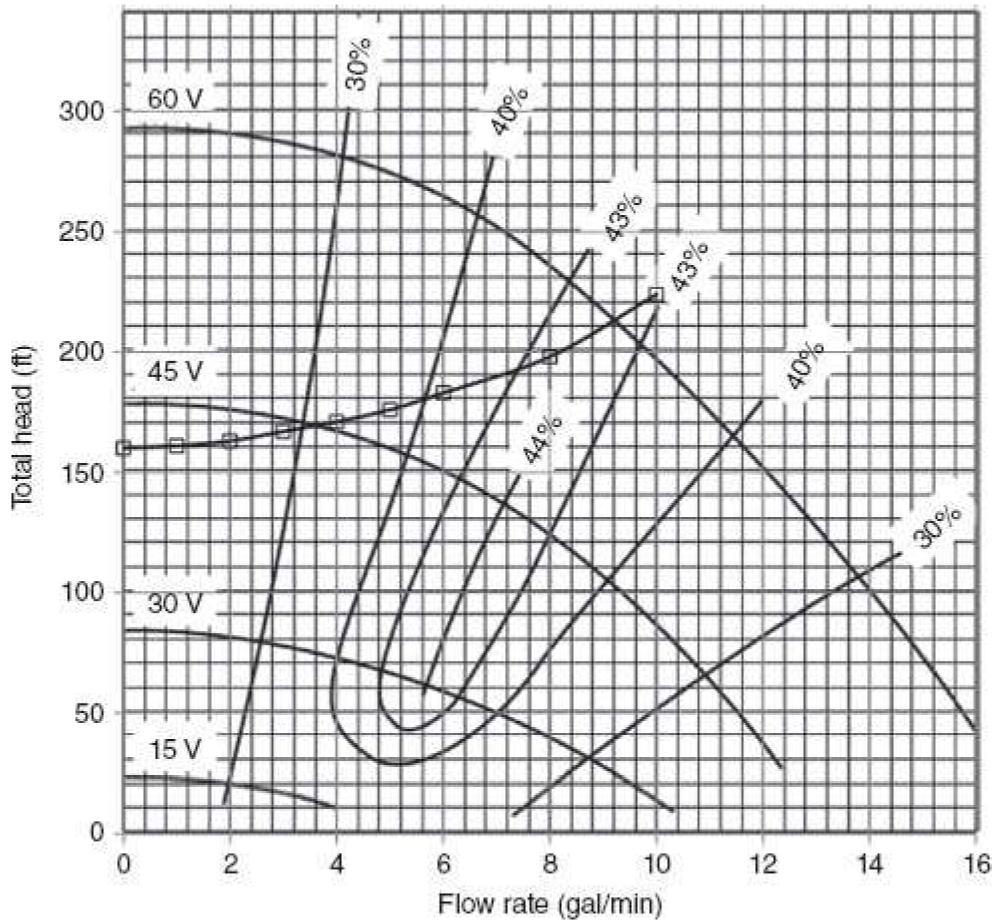
FIGURE 6.45 The hydraulic system curve for Example 6.16.



6.6.4 Putting it All Together to Predict Performance

Just as an *I–V* curve for a PV load is superimposed onto the *I–V* curves for the PVs, so too is the *Q–H* system curve superimposed onto the *Q–H* pump curve to determine the hydraulic operating point. For example, superimposing the system curve of Figure 6.45 onto the pump curves in Figure 6.42 gives us the diagram in Figure 6.46. A glance at the figure tells us a lot. For example, this pump will not deliver any water unless the voltage applied to the pump is at least about 40 V. At 45 V, about 3.2 gal/min would be pumped, while at 60 V the flow would be about 9.2 gal/min.

FIGURE 6.46 The system curve for Example 6.17, 150-ft well, superimposed onto the pump curves for the Jacuzzi SJ1C11. No flow occurs until pump voltage exceeds about 40 V.



What started out to be the very simplest physical combination of a PV module directly connected to a DC pump has turned into a quite complex, but fascinating, process in which a combination of nonlinear curves are laid one on top of another to try to predict hour-by-hour pumping rates. On the electrical side, by superimposing the pump curve onto the time-varying PV *I–V* curves, we can find hour-by-hour voltages delivered to the pump. By superimposing the hydraulic system curve onto the pump curves that vary with voltage, we can find the hour-by-hour pumping rates.

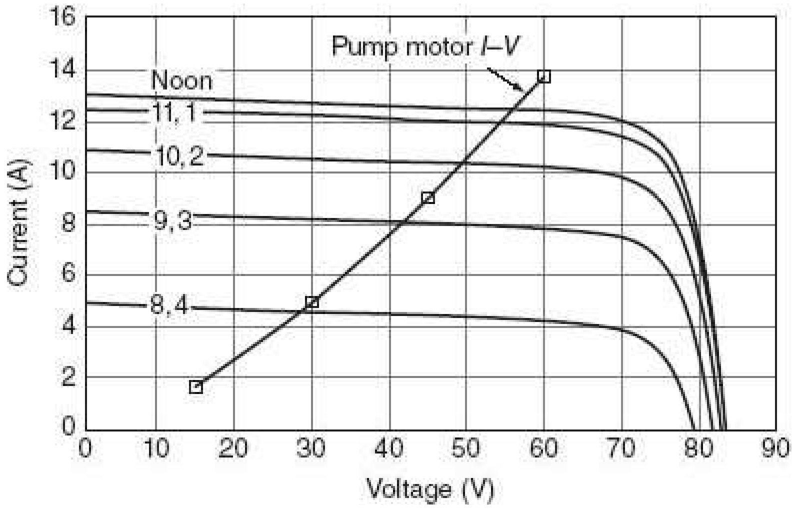
Let us illustrate the process with an example.

Example 6.17 Estimating Total Gallons Per Day Pumped. Pick a solar collector and use it to estimate the daily water pumped using the Jacuzzi SJ1C11 pump to deliver water from the 150-ft well analyzed in Example 6.16. The pump I–V curve is given in Figure 6.43. Design for a clear day in December at latitude 20° N with a south-facing 20° tilt array.

Solution. Figure 6.46 suggests that we will need a PV array that can deliver at least 40 V for much of the day and a glance at Figure 6.43 says it will need to supply over 8 A during that time. A combination of any of the collectors described in Table 5.3 can be used, but the simplest system will be one with the fewest modules, so let us try the SunPower E20/435 with V_{MPP} = 72.9 V, I_{MPP} = 5.97 A. The voltage is plenty for this task, but we will need two modules in parallel to get into the right current range.

From Appendix D, we get the hourly irradiance values shown below. By scaling the 1-sun short-circuit current for a pair of modules in parallel (I_{SC} = 6.43 A × 2 at 1000 W/m^2), we get the short-circuit currents at each hour. Then, all that we need to do is for each hour slide the 1-sun I–V curve down to match the I_{SC} for that hour. Adding the I–V curve for the pump motor from Figure 6.43 results in the following figure.

Two Parallel Modules 1-sun I_{SC} = 12.86 A									
Time	8	9	10	11	Noon	1	2	3	4
Insolation W/m^2	393	645	832	949	988	949	332	645	393
I_{SC} @ Insolation	5.1	8.3	10.7	12.2	12.7	12.2	10.7	8.3	5.1



From the figure, we can write down hourly voltages, which when coupled with the hydraulic curves in Figure 6.46 leads to the following table:

Time	8	9	10	11	Noon	1	2	3	4	Total gallons
Voltage	29	42	49	54	57	54	49	42	29	
Gal/min	0	0.5	7.5	7.8	8	7.8	7.5	0.5	0	
Gal/h	0	30	450	468	480	468	450	30	0	2376

So, our final design is two 435-W modules in parallel, directly coupled to the pump that should deliver close to 2400 gal over the course of one clear day.